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A CLSVOF method for simulating bubble motion in ultrasonic fields near a deformable solid cell

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ABSTRACT

An extended coupled level-set and volume-of-fluid (CLSVOF) method is developed for capturing compressible multiphase flow behavior induced by ultrasonic excitation, incorporating viscoelastic solid deformation effects. The proposed model ensures accurate mass conservation for compressible bubbles by introducing a mass-conservative formulation and effectively captures solid deformation by employing a fully Eulerian approach under axisymmetric coordinate system. The accuracy of the proposed method was verified through a series of benchmark tests, demonstrating close consistency with the reference results. Ultrasound-driven single-bubble growth and collapse were simulated, and the numerical predictions were validated against the Keller–Miksis analytical solution. The numerical model was also applied to investigate two-bubble dynamics, exhibiting mirror-like symmetric flow fields analogous to bubble–wall interactions. In both single- and two-bubble simulations, the results demonstrated excellent mass conservation throughout the entire simulation process. Finally, the CLSVOF method was applied to simulate ultrasound-driven bubble motion near a viscoelastic spherical solid cell. A parametric analysis was performed by varying the initial surface-to-surface distance and elastic shear modulus to evaluate their impact on bubble–cell interactions.

1. Introduction

Ultrasound-induced bubble dynamics play a critical role in diverse engineering applications, including water decontamination [1], drug delivery [2], and surface cleaning [3]. During bubble collapse, the resulting liquid jet can destroy organic matter such as viruses [1]; bubble expansion and contraction exert mechanical stresses that enhance drug uptake in adjacent tissues [2]; and liquid-jet-induced microstreaming imposes shear forces to biofilms attached to underlying substrates, thereby facilitating contaminant removal [3]. Although understanding bubble dynamics near deformable solids under ultrasonic fields is important, a comprehensive predictive framework for these phenomena has yet to be fully established.

Numerous numerical studies have extended the level-set (LS) method for simulating compressible gas-liquid interfacial dynamics [4–7]. Hu and Khoo [5] investigated bubble collapse under an incident shock, employing the LS approach in conjunction with a modified ghost fluid method (GFM) [4] to handle interfacial discontinuities caused by strong shocks. Huber et al. [6] simulated bubble oscillations in ultrasonic fields with a semi-implicit compressible projection framework that integrates the LS and GFM techniques, and validated their results against 1D Rayleigh–Plesset (RP) solutions. Lee and Son [7] developed an LS-based formulation integrating a semi-implicit pressure correction and the GFM to mitigate time-step constraints and accurately resolve

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interfacial discontinuities. However, these LS-based numerical approaches are known to suffer from mass loss due to their inherently non-conservative nature.

In parallel, a number of numerical investigations have utilized the volume-of-fluid (VOF) approach [8–10] for simulating compressible gas–liquid interface dynamics, benefiting from its inherent conservation of phase volume. Miller et al. [8] simulated underwater explosions using a VOF method within the OpenFOAM framework, where the gas and liquid phases were treated under isentropic and constant-temperature assumptions, respectively. Koch et al. [9] modeled laser-induced cavitation bubble dynamics under adiabatic conditions by reducing numerical errors associated with the additional compressibility term in the VOF advection equation. However, these conventional pressure-based VOF approaches are limited in simulating compressible bubble dynamics in acoustic fields involving negative pressures and exhibit significant mass loss because their compressible formulation is non-conservative [8–10].

To address these issues, several researchers [11,12] have developed mass-conservative VOF methods by coupling the volume and mass fluxes. Fuster and Popinet [11] proposed a projection-based VOF formulation applicable across all Mach number regimes within the Basilisk framework, wherein conservative quantities are advected based on the volumetric flux reconstructed at the interface. Saade et al. [12] extended this approach by incorporating thermal diffusion to study the response of an argon bubble under acoustic excitation. Despite these advancements, VOF-based numerical schemes still face fundamental challenges in accurately evaluating interfacial normals and curvatures, particularly for collapsing bubbles that undergo severe interface distortion under under-resolved conditions [13,14].

Several numerical studies [15–17] have extended the CLSVOF scheme, which couples the VOF and LS methods to maintain volume conservation and geometric fidelity, for analyzing compressible gas–liquid interactions. developed a compressible CLSVOF approach in OpenFOAM by coupling the LS method with an algebraic VOF scheme through additional terms accounting for compressibility, and applied it to calculate the oscillating water column problem. Chakraborty [17] extended the CLSVOF method, which couples a geometrically volume-conserving VOF scheme with the LS method, to simulate weakly compressible two-phase flows. They computed axisymmetric single-bubble oscillations under acoustic excitation below a pulse amplitude of 1 atm and evaluated their consistency against the prediction of the Rayleigh–Plesset (RP) equation. As far as the authors are aware, no CLSVOF formulation has yet been developed to employ a mass-conservative formulation for simulating bubble growth and collapse under ultrasonic conditions involving significant negative pressures.

Numerical investigations [18–20] on compressible multiphase flows involving deformable solids have primarily adopted Lagrangian approaches for the solid phase, where elastic stresses are evaluated based on the deformation of material markers distributed within the solid domain. Chahine et al. [18] performed numerical analyses of liquid jet impact and secondary toroidal bubble collapse to examine their effects on the plastic deformation of flexible solids, using a coupled fluid–solid framework in which the solid response was evaluated through a finite element formulation. Firly et al. [20] conducted numerical simulations of bubble collapse triggered by shock waves and the subsequent deformation of various metallic and polymeric materials by employing a VOF approach formulated on the conservative Euler equations and coupled with an arbitrary Lagrangian–Eulerian scheme [21] to describe solid deformation. Although Lagrangian methods are widely used for solid deformation analysis, mesh distortion caused by large deformations often necessitates remeshing [22,23].

To overcome this limitation, full Eulerian approaches [24–26] have been proposed to compute both solid and fluid phases simultaneously within a single Eulerian framework, thereby eliminating the need for mesh regeneration. Koukas et al. [25] extended a diffuse-interface scheme based on the six-equation model, including isotropic elastic solid deformation within an Eulerian framework, and simulated shock-induced bubble collapse near flexible biological tissues. Park and Son [26] developed an LS scheme featuring a sharply defined interface to simulate ultrasonic bubble dynamics that accounts for deformable solid behavior, and analyzed the influences of material properties, including acoustic parameters and shear modulus, on the large deformation of viscoelastic tissue. However, the CLSVOF method has not been extended to model bubble dynamics induced by ultrasound and the resulting viscoelastic solid deformation.

In this work, our previously developed CLSVOF method [27] was extended to numerically simulate bubble dynamics induced by ultrasound and the resulting deformation of adjacent solids by solving a mass-conservative formulation and incorporating the full Eulerian approach introduced in Ref. [28]. The van der Waals (VDW) and Tait equations were employed for the gas and the liquid/solid phases, assuming isothermal and isentropic conditions, respectively. Several benchmark tests were first conducted for validation, and the numerical results for single-bubble expansion under ultrasonic conditions were compared with the Keller–Miksis (KM) equation solution and results obtained using commercial software. The two-bubble dynamics subjected to ultrasonic excitation was also compared with single-bubble behavior with bubble–wall interactions. Finally, ultrasonic bubble motion coupled with viscoelastic cell deformation was simulated, and bubble–cell interactions were examined with respect to the elastic shear modulus and the initial surface-to-surface distance.

2. Numerical analysis

Building upon the CLSVOF framework proposed in our earlier work [27], the present study broadens the scheme to simulate ultrasound-induced bubble dynamics in a deformable viscoelastic solid. The developed numerical approach was verified through a series of test cases and then applied to compute bubble dynamics near a viscoelastic spherical cell subjected to ultrasonic pulse excitation, as shown in Fig. 1(a). Two LS functions [29,30], $\phi_{s/b}$, defined as signed distance functions from the water–solid and water–bubble interfaces, respectively, together with two VOF functions, $\alpha_{s/b}$, were employed to capture interfacial dynamics. In this study, all phases were treated as compressible and isothermal, and material properties were assumed constant except for the

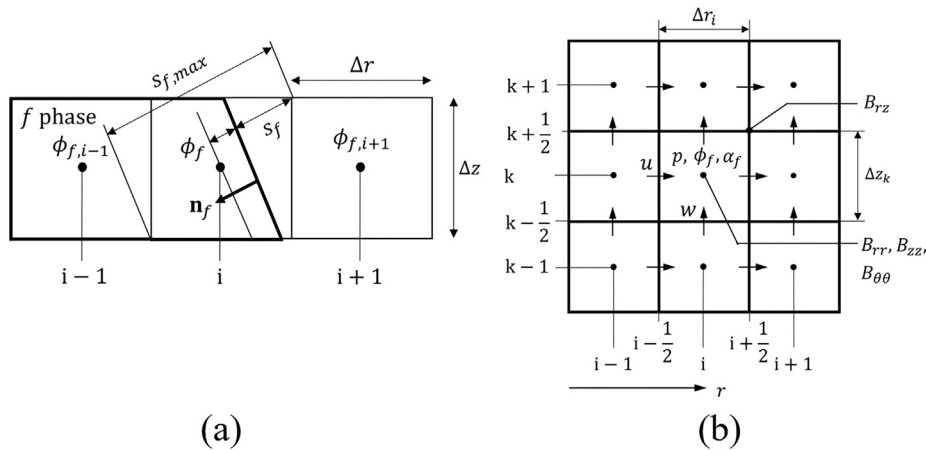


Fig. 1. Schematics of (a) the interface configuration, where the domain for phase f is enclosed by thick solid lines, and (b) the staggered mesh arrangement in axisymmetric coordinates.

densities. Thermal effects such as collapse-induced heating and phase-change heat transfer were not explicitly resolved under the present isothermal assumption, which has also been adopted in several previous numerical studies of bubble collapse [16,31–33]. For the microscale bubble considered in the present study, the thermal diffusion time scale is sufficiently short due to the small characteristic length scale. In addition, thermal diffusion and mass-transfer effects become less significant under high-frequency ultrasonic excitation [34]. Therefore, the present bubble–cell interaction is considered to be governed primarily by rapid mechanical dynamics rather than by thermal effects. The compressible bubble pressure p_b was evaluated under isothermal conditions using the VDW equation to accurately represent the elevated internal pressure generated during bubble collapse [35]:

$$p_b = (p_\infty + c_1 \rho_{b,\infty}^2) \frac{(\rho_{b,\infty}^{-1} - c_2)}{(\rho_b^{-1} - c_2)} - c_1 \rho_b^2 \quad (1)$$

where

$$c_1 = \frac{27}{64} \frac{R_g^2 T_c^2}{p_c}, \quad c_2 = \frac{R_g T_c}{8 p_c} \quad (2)$$

Here, $R_g = 287 \text{ J/kgK}$, and the subscript c represents the critical state. To account for the influences of the ultrasound pulse and the resulting shock emissions in both the solid and the surrounding liquid water domain, the pressures within the compressible solid and liquid phases, $p_{s/l}$, were determined using the Tait equation [26]:

$$p_{l/s} = p_{l/s,\infty} + \Pi_{l/s} \left[\left(\frac{\rho_{l/s}}{\rho_{l/s,\infty}} \right)^{\gamma_{l/s}} - 1 \right] \quad (3)$$

Here, $\Pi_{l/s}$ is a parameter related to the bulk modulus, defined as $\gamma_{l/s} \Pi_{l/s}$, where $\gamma_{l/s} = 7.15$. The sound speeds for all phases, $a_{b/l/s}$, were then derived as

$$a_b^2 = \frac{p_b + c_1 \rho_b^2}{\rho_b (1 - \rho_b c_2)} - 2 c_1 \rho_b \quad (4)$$

$$a_{l/s}^2 = \frac{\gamma_{l/s} (p_{l/s} - p_{l/s,\infty} + \Pi_{l/s})}{\rho_{l/s}} \quad (5)$$

2.1. Governing equations for compressible multiphase flow

In the bubble, liquid, and solid phases, the conservation equations can be written as

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\mathbf{u}_f \rho_f) = 0 \quad (6)$$

$$\frac{\partial \rho_f \mathbf{u}_f}{\partial t} + \nabla \cdot (\mathbf{u}_f \rho_f \mathbf{u}_f) = -\nabla p_f + \nabla \cdot \boldsymbol{\tau}_f \quad (7)$$

where

$$\boldsymbol{\tau}_{b/l} = \mu_{b/l} \left(\nabla \mathbf{u}_{b/l} + (\nabla \mathbf{u}_{b/l})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}_{b/l}) \mathbf{I} \right) \quad (8)$$

$$\boldsymbol{\tau}_s = \mu_s \left(\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T - \frac{2}{3} (\nabla \cdot \mathbf{u}_s) \mathbf{I} \right) + \boldsymbol{\tau}_e \quad (9)$$

Here, the subscript f indicates each phase: the solid (s) for $\phi_s \geq 0 \cap \phi_b < 0$, the liquid (l) for $\phi_s < 0 \cap \phi_b < 0$, and the bubble (b) for $\phi_s < 0 \cap \phi_b \geq 0$.

For the solid-gas-liquid multiphase domains, Eq. (7) can be reformulated in the following form:

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u} \mathbf{u} = -(\nabla p + \sigma \kappa_l \nabla H_b) + \nabla \cdot \alpha_s \boldsymbol{\tau}_e + \nabla \cdot \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \quad (10)$$

where

$$\rho = \rho_s \alpha_s + [\rho_b \alpha_b + \rho_l (1 - \alpha_b)] (1 - \alpha_s) \quad (11)$$

$$\mu = \mu_s \alpha_s + [\mu_b \alpha_b + \mu_l (1 - \alpha_b)] (1 - \alpha_s) \quad (12)$$

$$\kappa_l = \nabla \cdot \frac{\nabla \phi_b}{|\nabla \phi_b|} \quad (13)$$

$$H_b = 1 \quad \text{if } \phi_b \geq 0 \\ = 0 \quad \text{if } \phi_b < 0 \quad (14)$$

$$\boldsymbol{\tau}_e = G (\mathbf{B} - \mathbf{I}) \quad (15)$$

Here, κ_l denotes the bubble–water interfacial curvature. H_b represents the sharp Heaviside function, and $\boldsymbol{\tau}_e$, $\mathbf{B}$, and G denote the neo-Hookean elastic stress, the spatial deformation tensor defined as $\mathbf{B} = (\partial \mathbf{x} / \partial \mathbf{X})(\partial \mathbf{x} / \partial \mathbf{X})^T$, and the shear modulus, respectively. In the Eulerian framework, the evolution equation for $\mathbf{B}$ is expressed as follows [28,36]

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = (\nabla \mathbf{u})^T \cdot \mathbf{B} + \mathbf{B} \cdot (\nabla \mathbf{u}) \quad (16)$$

2.2. Mass-conserving CLSVOF method

To ensure mass conservation, the VOF functions were used to correct the interfaces defined by the LS functions [27,37]. The LS and VOF functions were then advected successively in the r - and z - directions as follows:

$$\frac{\partial \phi_f}{\partial t} + \mathbf{u}_f \cdot \nabla \phi_f = 0 \quad (17)$$

$$\frac{\partial \rho_f \alpha_f}{\partial t} + \nabla \cdot \rho_f \alpha_f \mathbf{u}_f = 0 \quad (18)$$

Here, accurate evaluation of the volume flux in the convective term of Eq. (18) requires interface reconstruction based on the advanced α_f and the interfacial normal $\mathbf{n}_f = \nabla \phi_f / |\nabla \phi_f|$. It should be noted that $\mathbf{n}_f$ can be calculated more straightforwardly using ϕ_f , which is updated by Eq. (17). Eq. (18) satisfies the mass conservation property, with the density ρ_f obtained from the mass Eq. (6).

2.3. Discretization of the CLSVOF method

The temporally discretized formulation of the mass Eq. (6) can be written as follows:

$$\frac{\rho_f^{n+1} - \rho_f^n}{\Delta t} + \nabla \cdot (\mathbf{u}_f \rho_f)^n = 0 \quad (19)$$

Here, the convective term in the mass Eq. (19) was evaluated using the ghost fluid method (GFM) [4,7], which effectively incorporates discontinuous interfacial conditions. The VOF advection Eq. (18) was computed using an operator-splitting approach [27,37], with ρ_f obtained from Eq. (19). The advection process was further decomposed into two fractional steps along the radial and z -axis directions as

$$\frac{\rho_f^n \alpha_f^* - \rho_f^n \alpha_f^n}{\Delta t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_f u_f \alpha_f^n) = \frac{\alpha_f^*}{r} \left(\frac{\partial r \rho_f u_f}{\partial r} \right)^n \quad (20)$$

$$\frac{\rho_f^n \alpha_f^{**} - \rho_f^n \alpha_f^*}{\Delta t} + \frac{\partial}{\partial z} (\rho_f w_f \alpha_f^*) = \alpha_f^* \left(\frac{\partial \rho_f w_f}{\partial z} \right)^n \quad (21)$$

$$\frac{\rho_f^{n+1} \alpha_f^{n+1} - \rho_f^n \alpha_f^{**}}{\Delta t} = -\alpha_f^* \left(\frac{1}{r} \frac{\partial r \rho_f u_f}{\partial r} + \frac{\partial \rho_f w_f}{\partial z} \right)^n \quad (22)$$

Here, the source terms in Eqs. (20) and (21) preserves the invariance of α_f in single-phase regions ($\alpha_f = 1 \cup 0$) [27]. The volumetric flux at the cell faces, which appears as the advection terms in Eqs. (20) and (21), corresponds to the volume swept by the interface

during Δt in each coordinate direction. This swept volume is explicitly determined from the interfacial normal $\mathbf{n}_f$ and the distance s_f from a vertex to the interface [27], as illustrated in Fig. 1(a). The variables $\mathbf{n}_f$ and s_f take the following functional forms:

$$\mathbf{n}_f = \frac{\nabla \phi_f}{|\nabla \phi_f|} \quad (23)$$

$$s_f^n = f(\phi_f^n, \alpha_f^n); \quad s_f^* = f(\phi_f^*, \alpha_f^*) \quad (24)$$

Here, the distance s_f explicitly determines the volumetric flux together with the interfacial normal $\mathbf{n}_f$, requiring almost no iteration [27]. Further details of the formulation used to evaluate the volumetric flux and s_f can be found in our previous studies [27]. The advection equation of ϕ_f in Eqs. (23)–(24) is directionally decomposed and advanced in conjunction with Eqs. (20) and (21) as follows:

$$\frac{\phi_f^* - \phi_f^n}{\Delta t} + \frac{u_f^n}{r} \frac{\partial r \phi_f^n}{\partial r} = 0 \quad (25)$$

$$\frac{\phi_f^{**} - \phi_f^*}{\Delta t} + w_f^n \frac{\partial \phi_f^*}{\partial z} = 0 \quad (26)$$

Here, the convective terms in Eqs. (25) and (26) were spatially discretized through a second-order finite-volume formulation that possesses the total variation diminishing (TVD) property with the monotized central (MC) flux limiter [38,39]. Since the advected ϕ_f generally loses its signed-distance property, ϕ_f was reinitialized to restore its distance-field character based on the reconstructed interface, as follows:

$$\phi_f^{n+1} = s_f^{**} - \frac{1}{2} s_{f,max}^{**} \quad \text{if } 0 < \alpha_f^{n+1} < 1 \quad (27)$$

$$\frac{\partial \phi_f}{\partial \tau^*} = S(\phi_f)(1 - |\nabla \phi_f|) \quad (28)$$

where

$$s_f^{**} = f(\phi_f^{**}, \alpha_f^{n+1}) \quad (29)$$

$$s_{f,max}^{**} = \Delta r |n_{f,r}^{**}| + \Delta z |n_{f,z}^{**}| \quad (30)$$

$$\begin{aligned} S(\phi_f) &= 0 & \text{if } |\phi_f| \leq \frac{h}{2} \\ &= \phi_f / \sqrt{\phi_f^2 + h^2} & \text{if } |\phi_f| > \frac{h}{2} \end{aligned} \quad (31)$$

Here, ϕ_f in the vicinity of the interface ($0 < \alpha_f^{n+1} < 1$), was first reinitialized directly from the geometric relation in Eq. (27), as shown in Fig. 1(a). The LS function ϕ_f was then reinitialized iteratively solving Eq. (28) with an artificial time step τ^* . The signed function $S(\phi_f)$ was introduced to improve volume conservation by fixing ϕ_f near the interface ($|\phi_f^{n+1}| \leq h/2$) [26].

2.4. Discretization of the conservation equations

A staggered mesh configuration was employed for the spatial discretization. The convective fluxes were evaluated through a second-order TVD scheme with the monotized central (MC) flux limiter [38], whereas the diffusive fluxes were approximated by a second-order central differencing. Temporal integration was performed in a fractional-step manner.

$$\rho^{n+1} \left(\frac{\mathbf{u}^* - \mathbf{u}_f^n}{\Delta t} + \mathbf{u}_f^n \nabla \mathbf{u}_f^n \right) = -(\nabla p + \sigma \kappa_l \nabla H_b)^n + \nabla \cdot \mu^{n+1} \nabla \mathbf{u}^* + \rho^{n+1} \mathbf{g} + \mathbf{f}^n + \nabla \cdot \alpha_s \boldsymbol{\tau}_e \quad (32)$$

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \frac{\Delta t}{\rho^{n+1}} [(\nabla p + \sigma \kappa_l \nabla H_b)^{n+1} - (\nabla p + \sigma \kappa_l \nabla H_b)^n] \quad (33)$$

$$\nabla \cdot \mathbf{u}^{n+1} = -\frac{H_f}{\rho_f \alpha_f^2} \frac{p^{n+1} - p_f^*}{\Delta t} \quad (34)$$

where

$$\rho^{n+1} = \rho(\rho_f^{n+1}, \alpha_f^{n+1}); \quad \mu^{n+1} = \mu(\mu_f, \alpha_f^{n+1}); \quad (35)$$

$$\mathbf{f} = \nabla \cdot \mu \left[(\nabla \mathbf{u})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right] \quad (36)$$

$$p_f^* = p_f(\rho_f^{n+1}) \quad (37)$$

Here, the density ρ^{n+1} in Eqs. (32) and (33) was obtained by spatial linear interpolation using Eq. (35), based on ρ_f^{n+1} advanced by Eq. (19), and the weighted combination of the phase densities as given in Eq. (11). The momentum equation is discretized in

a non-conservative form to alleviate numerical instabilities associated with abrupt density variations near the interface. Similar approaches have also been employed in previous numerical studies of compressible multiphase flows [40–43]. In Eq. (33), the pressure p^{n+1} used to correct $\mathbf{u}^*$ was calculated from Eq. (34), where the intermediate pressure p_f^* was obtained from Eq. (37) using ρ_f^{n+1} . This semi-implicit pressure-correction procedure has been widely employed in previous numerical studies to alleviate the acoustic time-step restriction in weakly compressible media under ultrasonic excitation [7,26,44,45]. Eq. (34) is obtained from the phase-wise continuity equation combined with the corresponding equation of state under the sharp-interface assumption. The velocity divergence is related to the pressure variation through the phase-dependent sound speed a_f , where $a_f^2 = \partial p_f / \partial \rho_f$. The resulting formulation is generic and applicable to all phases, while the specific equation of state differs for each phase. The pressure p_f^* was evaluated using the VDW and Tait equations, expressed by Eqs. (1) and (3), respectively.

2.5. Detailed discretization of $\mathbf{B}$

The transport Eq. (16) with $\tilde{\mathbf{B}} = \mathbf{B} - \mathbf{I}$ was temporally discretized as follows [35,36]:

$$\frac{\tilde{\mathbf{B}}^{n+1} - \tilde{\mathbf{B}}^n}{\Delta t} + \mathbf{u}^n \cdot \nabla \tilde{\mathbf{B}}^n = (\nabla \mathbf{u}^n)^T \cdot \tilde{\mathbf{B}}^n + \tilde{\mathbf{B}}^n \cdot (\nabla \mathbf{u}^n) + (\nabla \mathbf{u}^n)^T + (\nabla \mathbf{u}^n) \quad (38)$$

For the axisymmetric coordinate system,

$$\frac{\tilde{B}_{rr}^{n+1} - \tilde{B}_{rr}^n}{\Delta t} + u^n \frac{\tilde{B}_{rr}^n}{\partial r} + w^n \frac{\tilde{B}_{rr}^n}{\partial z} = 2\tilde{B}_{rr}^n \left(\frac{\partial u}{\partial r} \right)^n + 2\tilde{B}_{rz}^n \left(\frac{\partial u}{\partial z} \right)^n + 2 \left(\frac{\partial u}{\partial r} \right)^n \quad (39)$$

$$\frac{\tilde{B}_{rz}^{n+1} - \tilde{B}_{rz}^n}{\Delta t} + u^n \frac{\tilde{B}_{rz}^n}{\partial r} + w^n \frac{\tilde{B}_{rz}^n}{\partial z} = \tilde{B}_{rr}^n \left(\frac{\partial w}{\partial r} \right)^n + \tilde{B}_{zz}^n \left(\frac{\partial u}{\partial z} \right)^n + \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial y} \right)^n \quad (40)$$

$$\frac{\tilde{B}_{zz}^{n+1} - \tilde{B}_{zz}^n}{\Delta t} + u^n \frac{\tilde{B}_{zz}^n}{\partial r} + w^n \frac{\tilde{B}_{zz}^n}{\partial z} = 2\tilde{B}_{zz}^n \left(\frac{\partial w}{\partial z} \right)^n + 2\tilde{B}_{rz}^n \left(\frac{\partial w}{\partial r} \right)^n + 2 \left(\frac{\partial w}{\partial z} \right)^n \quad (41)$$

$$\frac{\tilde{B}_{\theta\theta}^{n+1} - \tilde{B}_{\theta\theta}^n}{\Delta t} + u^n \frac{\tilde{B}_{\theta\theta}^n}{\partial r} + w^n \frac{\tilde{B}_{\theta\theta}^n}{\partial z} = 2 \left(\frac{u}{r} \right)^n (\tilde{B}_{\theta\theta}^n + 1) \quad (42)$$

Here, $\tilde{B}_{rz}$ denotes the off-diagonal component located at $(i+1/2, k+1/2)$, whereas the diagonal components $\tilde{B}_{rr/zz/\theta\theta}$ are positioned at (i, j) , as illustrated in Fig. 1(b). An additional diagonal component $B_{\theta\theta}$ and its transport Eq. (42) are required under the axisymmetric coordinate system, which are absent in the 2D Cartesian coordinate system. Using the updated $\tilde{\mathbf{B}}$, the elastic stress terms $\nabla \cdot \boldsymbol{\tau}_e$ in the Eq. (32) is spatially discretized for axisymmetric coordinates as:

$$(\nabla \cdot \boldsymbol{\tau}_e)_r = G \left[\frac{(r\tilde{B}_{rr})_{(i,k)} - (r\tilde{B}_{rr})_{(i-1,k)}}{r\Delta r_{i-\frac{1}{2}}} + \frac{\tilde{B}_{rz(i-\frac{1}{2},k+\frac{1}{2})} - \tilde{B}_{rz(i-\frac{1}{2},k-\frac{1}{2})}}{(\Delta z)_k} - \frac{\tilde{B}_{\theta\theta(i-\frac{1}{2},k)}}{r_{i-\frac{1}{2}}} \right] \quad (43)$$

$$(\nabla \cdot \boldsymbol{\tau}_e)_z = G \left[\frac{\tilde{B}_{zz(i,k)} - B_{zz(i,k-1)}}{\Delta z_{k-\frac{1}{2}}} + \frac{(r\tilde{B}_{rz})_{(i+\frac{1}{2},k-\frac{1}{2})} - (r\tilde{B}_{rz})_{(i-\frac{1}{2},k-\frac{1}{2})}}{r\Delta r_i} \right] \quad (44)$$

Here, no extra interpolation was required owing to the staggered definition of the deformation tensor components [36], as illustrated in Fig. 1(b). Note that, in Eq. (43), the elastic stress effects can become significant near the axis due to the denominator r in the term $(\tilde{B}_{\theta\theta}/r)_{i-1/2,k}$.

3. Results and discussion

3.1. Verification of the CLSVOF approach

3.1.1. Bubble rising problem

To verify the present numerical model for two-phase flow, a rising bubble test was performed and compared with previous numerical results [46] and the commercial software FLUENT, as shown in Fig. 2. The computational setup followed the benchmark conditions reported in Ref. [46]. Once released, the initially stationary bubble began to rise from its center at $t = 1.25$ s, and subsequently deformed into a crescent shape at $t = 2.50$ s. Strong surface-tension effects near the bubble edges, where the curvature is large, drew the interface inward toward the middle region at $t = 3.75$ s. The rising bubble exhibited highly unsteady and oscillatory motion, gradually settling into a cap shape at $t = 8.75$ s. As it continues to rise, the bubble became increasingly flattened and widened, forming a thin liquid film over its upper surface at $t = 9.75$ s until the film eventually ruptures.

The CLSVOF computations exhibited close consistency with FLUENT simulations employing a geometric PLIC–VOF scheme, as well as with data reported in Ref. [46]. The time-dependent behavior of the top-most bubble position $y_{b,top}$ also follows the same trend as the results from FLUENT and Ref. [46] throughout the rising process, as shown in Fig. 2(b). These results demonstrate that our current scheme can successfully capture the dynamic behavior of bubbles.

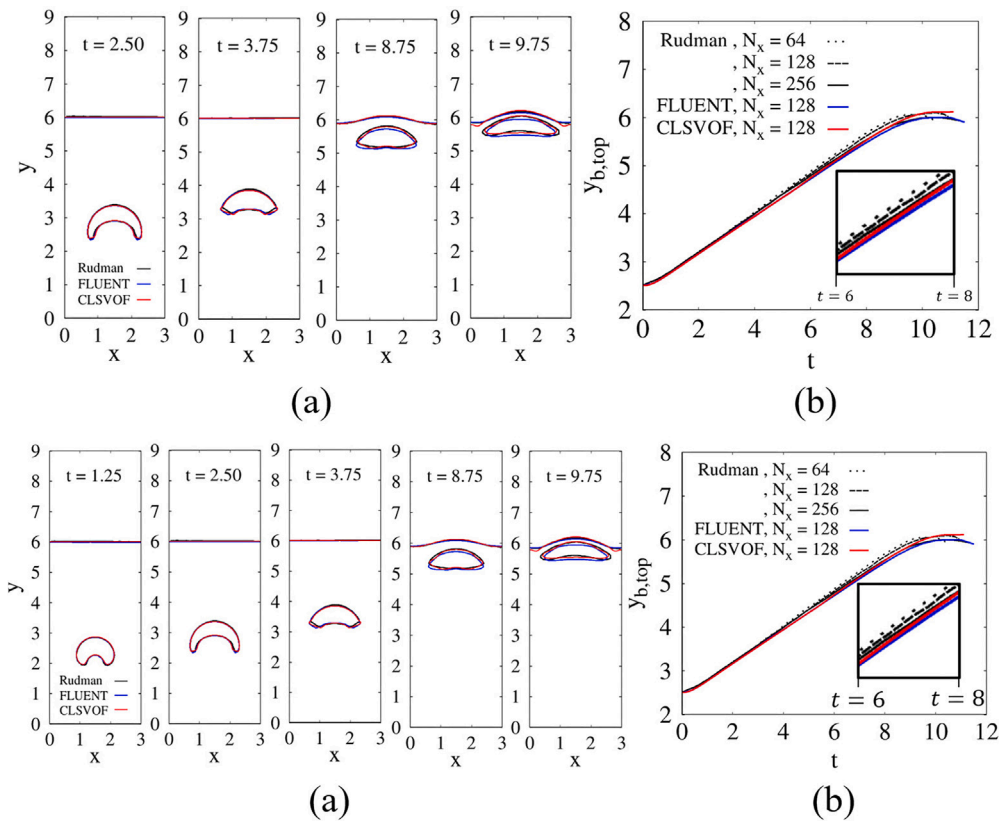


Fig. 2. Comparison of (a) bubble shapes and (b) top-surface positions of a rising bubble with the results of Rudman [46] and the commercial software. The inset in the lower right corner of (b) highlights the close agreement between the present numerical results and Rudman's solution.

3.1.2. Solid deformation in a lid-driven cavity

To verify the present CLSVOF framework for solid deformation, numerical simulations were performed for a compliant disk confined in a cavity with a movable top lid, a configuration commonly used in previous studies [22,28,47] employing either Lagrangian or Eulerian frameworks, as shown in Fig. 3. A neo-Hookean solid disk of radius 0.2 m was initially placed at the center (0.6, 0.5) m of a 2D cavity ($0 \leq x \leq 1$ m $\cap$ $0 \leq y \leq 1$ m) under stress-free conditions. The top lid moved in the x -direction at 1 m/s, while the remaining walls were stationary. The physical properties of the solid and fluid phases were set to $\rho_s = \rho_f = 1$ kg/m³, $\mu_s = \mu_f = 0.01$ Pa s, and $G = 0.1$ Pa or 10 Pa.

Fig. 3 presents the deformation behavior of the solid disk at three different grid resolutions ($h = 1/100$, $1/200$, and $1/400$ m). As the cavity flow developed with the motion of the top lid, the disks were advected along the flow field. The disk with $G = 0.1$ Pa underwent significant deformation under hydrodynamic traction, as shown in Fig. 3(a), and experienced the largest strain when approaching the cavity wall at $t = 4.69$ μ s, while avoiding overlap with the wall owing to the soft lubrication effect [28].

In contrast, the stiffer disk with $G = 10$ Pa retained an almost circular shape, as illustrated in Fig. 3(b), due to the relatively low hydrodynamic traction. The deformation patterns obtained for the two shear moduli show close agreement with the results reported by Zhao et al. [22], exhibiting little sensitivity to grid resolution.

3.1.3. Droplet equilibrium on surfaces under prescribed contact angle conditions

The present CLSVOF method was applied to simulate droplet equilibrium on surface boundaries with a constant contact angle condition. The simulations employed the material properties of a water–air system. All computations were carried out under zero-gravity conditions using a uniform grid with a spacing of $h = R_{do}/20$, where R_{do} denotes the initial droplet radius. The following condition was applied at the bottom wall or along the cylindrical solid surface:

$$\mathbf{c}_d \cdot \frac{\nabla \phi_d}{|\nabla \phi_d|} = 0 \quad (45)$$

where

$$\mathbf{c}_d = \mathbf{n}_{w/s} \sin \theta_c + \mathbf{t}_{w/s} \cos \theta_c \quad (46)$$

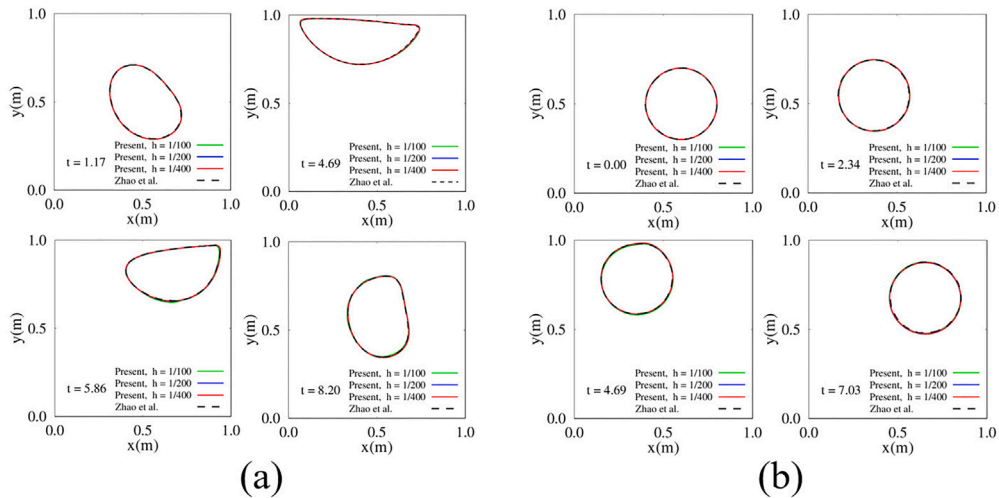


Fig. 3. Disk deformation in a lid-driven cavity for (a) $G = 0.1$ Pa and (b) $G = 10$ Pa, obtained with different mesh resolutions.

$$\mathbf{t}_{w/s} = \frac{\mathbf{n}_{w/s} \times \mathbf{a}}{|\mathbf{n}_{w/s} \times \mathbf{a}|}; \quad \mathbf{a} = \mathbf{n}_{w/s} \times \nabla \phi_d \quad (47)$$

where ϕ_d represents the LS field employed for capturing the droplet surface; $\mathbf{c}_d$ denotes the tangential vector at the droplet surface; $\mathbf{n}_{w/s}$ and $\mathbf{t}_{w/s}$ represent the normal and tangential vectors along the wall or curved solid boundary, respectively; and θ_c is the prescribed contact angle, ranging from 30° to 150° .

Fig. 4 compares the numerical results for droplet spreading on a wall boundary in an axisymmetric configuration with the corresponding analytical solution. The droplet interface reaches an equilibrium state when surface tension and wall adhesion forces are balanced, forming a truncated spherical that satisfies the following geometric relation [48]:

$$R_c = R_d \sin \theta_c; \quad h_d = R_d(1 - \cos \theta_c) \quad (48)$$

$$R_d = R_{do} \left(\frac{4}{(1 - \cos \theta_c)(1 + \sin^2 \theta_c - \cos \theta_c)} \right)^{1/3} \quad (49)$$

where R_d , h_d , and R_c denote the radius of curvature, droplet height, and contact radius corresponding to the equilibrium interfacial configuration, respectively, while the analytical spherical cap is centered at $(0, h_d - R_d)$. The numerical and analytical profiles for $\theta_c = 30^\circ$, 90° , and 150° exhibit very close agreement, as illustrated in Fig. 4(a). The normalized droplet height h_d/R_{do} and contact radius R_c/R_{do} are also compared with the analytical solution and with data from a previous study [48], as plotted in Fig. 4(b)–(c). These results demonstrate that the present model accurately predicts the equilibrium interface even under highly hydrophilic and hydrophobic wall conditions.

Fig. 5 compares the numerical results for droplet equilibrium on an immersed cylindrical solid with the exact analytical solution. The exact solution is expressed in terms of the equilibrium droplet radius R_d and the angles β_1 (or β_2), as illustrated in Fig. 5(a). These parameters are iteratively determined from the following geometrically derived nonlinear equations:

$$R_d = \sqrt{\frac{\pi R_{do}^2 + R_s^2(\beta_2 - \sin \beta_2 \cos \beta_2)}{\pi - (\beta_1 - \sin \beta_1 \cos \beta_1)}} \quad (50)$$

$$\tan \beta_1 = \frac{R_s \sin(\pi - \theta_c)}{R_d + R_s \cos(\pi - \theta_c)}; \quad \beta_1 + \beta_2 = \pi - \theta_c \quad (51)$$

Here, R_s denotes the radius of the cylindrical solid, and the equilibrium droplet resides at $z = z_{sc} + R_d \cos \beta_1 + R_s \cos \beta_2$, where z_{sc} is the vertical coordinate of the cylinder center. The two-dimensional droplet adhering to the circular cylinder becomes stationary after an initial oscillatory period due to viscous damping, and eventually converges to a truncated circular shape constrained by the solid surface, as illustrated in Fig. 5(b). Corresponding numerical predictions for the steady-state droplet interface show excellent agreement with the exact solutions even at $\theta_c = 30^\circ$ and 150° . These results confirm that the present numerical formulation can accurately maintain the prescribed contact angle on both highly hydrophilic and highly hydrophobic solid surfaces.

3.1.4. Impact of a cell-laden droplet on a liquid pool

For verification of the proposed CLSVOF approach in multiphase flow problems, simulations were performed for a droplet containing a solid particle impacting a liquid water pool. The results were compared with those reported in a previous study [47],

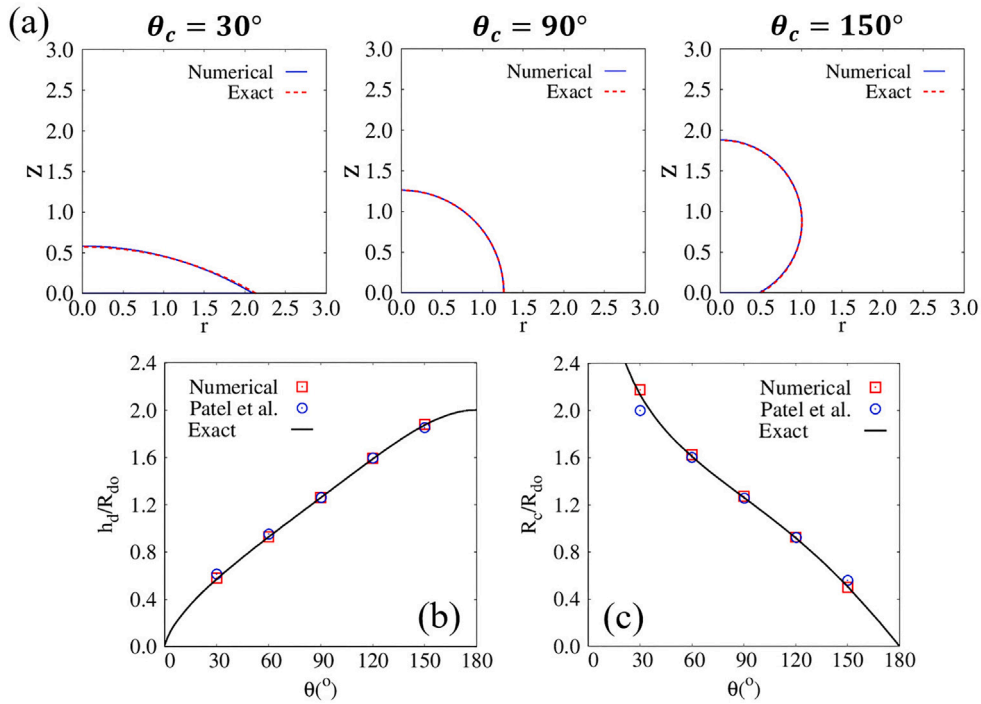


Fig. 4. Steady-state droplet on a flat surface: (a) equilibrium interfaces for $\theta_c = 30^\circ$, 90° , 150° ; (b) normalized droplet height h_d/R_{d0} ; and (c) normalized contact radius R_c/R_{d0} . The solid black line represents the analytical solution for the droplet height and contact radius.

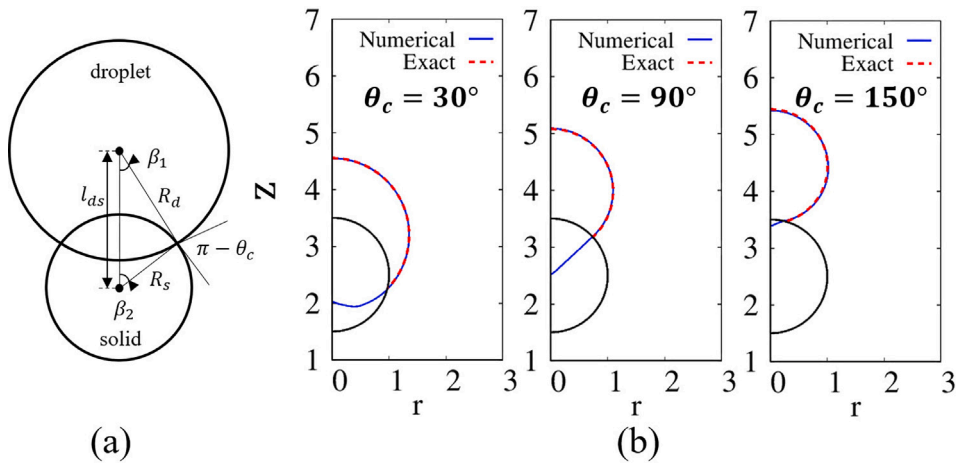


Fig. 5. Equilibrium interfacial shapes of droplets adhering to a cylindrical solid with contact angle θ_c : (a) schematic illustration and (b) numerical results for $\theta_c = 30^\circ$, 90° , and 150° .

as depicted in Fig. 6. The fluid properties correspond to a water–air system, while the immersed neo-Hookean solid sphere was modeled with $G = 10$ kPa, $\mu_s = \mu_w$, and $\rho_s = \rho_w$. The simulation was performed assuming axisymmetry, employing a uniform grid with a spatial resolution of $h = 0.5 \mu\text{m}$ within the computational domain $(0, 0) \leq (r, z) \leq (80, 180) \mu\text{m}$.

The initial droplet, with a radius of $R_{d0} = 30 \mu\text{m}$, was positioned in contact with the liquid pool at a height equal to R_{d0} above the pool surface, while a deformable solid sphere of radius $R_{s0} = 0.5R_{d0}$ was embedded inside the droplet. At the beginning of the simulation, the droplet merged with the pool surface, and surface tension induced a radial contraction that compressed the immersed elastic solid, as shown at $t = 9.21 \mu\text{s}$. The resulting capillary pressure drove a rapid downward motion of the solid, as observed at $t = 23.38 \mu\text{s}$.

After being released into the liquid pool, the solid sphere recovered its nearly spherical shape due to elastic restoration, while the surrounding liquid formed an elongated column along the z -axis. As the liquid column collapsed, it exerted a compressive force

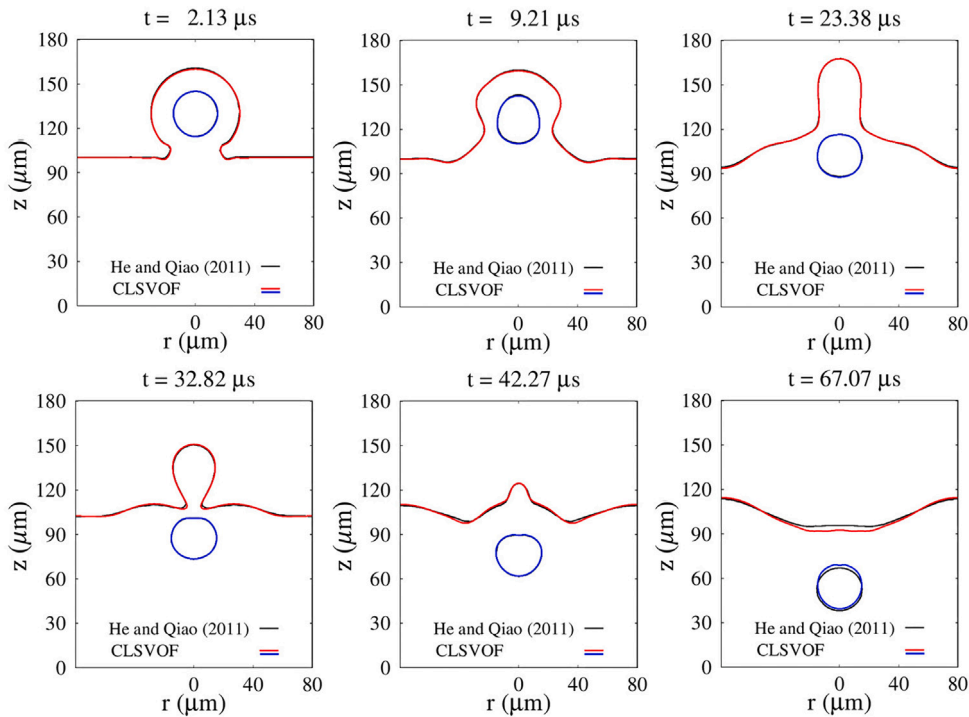


Fig. 6. Temporal evolution of air–water and water–solid interfaces during the coalescence of a cell-laden droplet with a liquid water pool. The black lines indicate the interfacial contours reported in Ref. [47], obtained using a Lagrangian mesh for the solid cell.

on the top surface of the solid, leading to further downward motion, as illustrated for $32.82 \mu\text{s} \leq t \leq 42.27 \mu\text{s}$. Subsequently, the solid continued to descend due to inertia and gradually returned to an equilibrium configuration at $t = 67.07 \mu\text{s}$. The overall dynamics of the air–water interface and the elastic cell show good agreement with the interfacial configurations reported in Ref. [47], where the solid deformation was computed using the ZFM method [22] on a Lagrangian mesh.

3.2. Compressible two-phase flow in ultrasonic fields

A stationary spherical air bubble immersed in quiescent water was simulated under axisymmetric conditions at $p_\infty = 1 \text{ atm}$ and $T_\infty = 20^\circ\text{C}$. The material properties of air and water were set as: $\sigma = 0.073 \text{ kg/s}^2$, $\Pi_l = 3.31 \times 10^8 \text{ Pa}$, $\mu_l = 10^{-3} \text{ Pa s}$, $\mu_b = 10^{-5} \text{ Pa s}$, $\rho_{l,\infty} = 998 \text{ kg/m}^3$, $\rho_{b,\infty} = 1.23 \text{ kg/m}^3$.

A microbubble with an initial radius $R_{b0} = 1 \mu\text{m}$ was placed more than $5890R_{b0}$ away from both the radial and bottom boundaries to minimize the influence of reflected pressure waves, following the setup of our previous study [26]. To impose the ultrasonic excitation, the upper boundary located $500R_{b0}$ above the bubble surface was prescribed with a pressure condition, expressed as follows:

$$p_u = p_\infty + P_u \sin(2\pi f_u t) \quad \text{if } f_u t \leq 1 \quad (52)$$

where P_u and f_u denote the pulse amplitude and frequency, respectively. The same ultrasonic conditions, $P_u = 0.3 \text{ MPa}$ and $f_u = 1 \text{ MHz}$, were applied to all simulation cases.

3.2.1. Ultrasound-driven single bubble motion

The proposed CLSVOF method was first applied in the analysis of a single air bubble subjected to an ultrasonic field. The resulting velocity and pressure fields in the surrounding liquid are presented in Fig. 7. The initially stationary bubble began to expand after $t > 0.32 \mu\text{s}$, when the acoustic wave propagated through the surrounding water and reached the bubble. As the bubble grew during the negative-pressure phase, a radially expanding liquid flow developed around it, as observed at $t = 0.57 \mu\text{s}$. When the ultrasonic wave entered the compression phase ($t > 0.82 \mu\text{s}$), the bubble, driven by liquid inertia, kept enlarging until it attained the maximum size at $t = 0.92 \mu\text{s}$, followed by a subsequent collapse.

During the collapse stage, a sink flow formed around the bubble, directed toward its center, resulting in elevated pressure in the surrounding liquid, as shown at $t = 1.133 \mu\text{s}$, which was also observed in previous numerical studies [49]. A slight asymmetry near the bubble surface induced a non-spherical collapse, emitting a shock wave and driving the bubble downward as it neared its minimum size, as illustrated at $t = 1.137 \mu\text{s}$. This behavior is consistent with earlier findings [16], which demonstrated that spatial

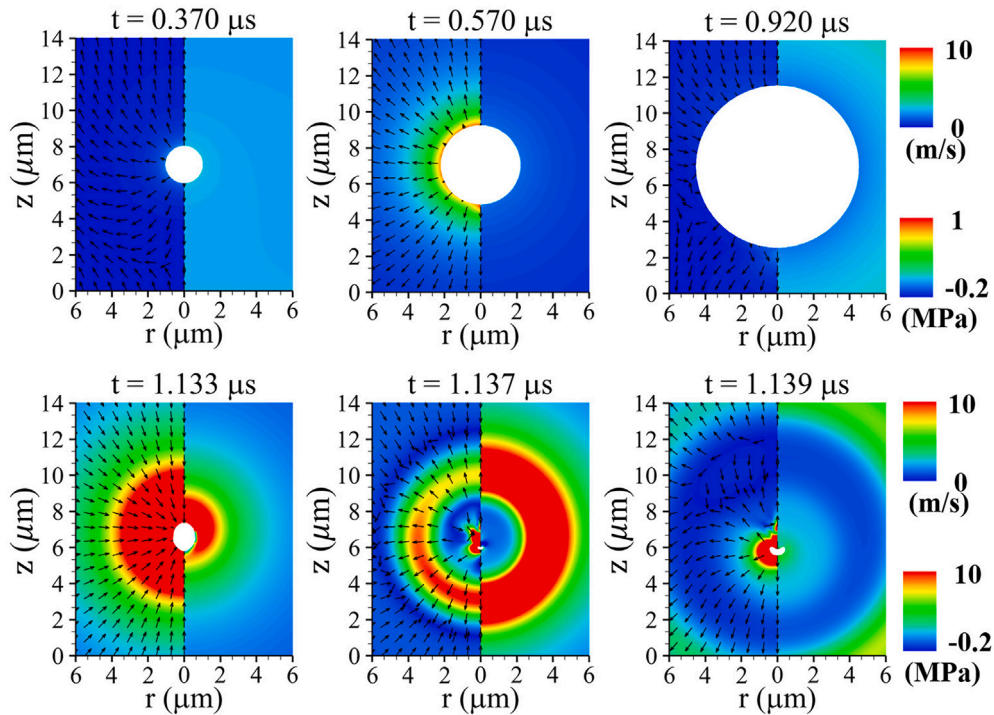


Fig. 7. Liquid velocity (left) and pressure (right) fields associated with ultrasonic bubble motion at $P_u = 0.3$ MPa and $f_u = 1$ MHz. The white region denotes the bubble interior.

variations in pressure and the propagation characteristics of the acoustic field strongly influence the development of non-spherical bubble dynamics, even when solid boundaries are absent.

However, the degree of asymmetry in the present flow field was insufficient to generate a strong liquid jet. Consequently, the bubble re-expanded into a mildly non-spherical shape without fragmentation, as observed at $t = 1.139 \mu\text{s}$. After re-expansion, the bubble gradually relaxed toward a spherical equilibrium state through a few oscillations driven by viscous damping and surface tension.

To evaluate the validity of the present CLSVOF formulation for compressible two-phase flow simulations, Fig. 8 compares the current results with those obtained using the commercial software FLUENT and with the numerical predictions from the corresponding KM equation [26]:

$$\left(1 - \frac{\dot{R}_b}{a_l}\right) R_b \ddot{R}_b + \frac{3}{2} \dot{R}_b^2 \left(1 - \frac{\dot{R}_b}{3a_l}\right) = \frac{1}{\rho_{l,\infty}} \left(1 + \frac{\dot{R}_b}{a_l} + \frac{R_b}{a_l} \frac{d}{dt}\right) (p_l + p_u) \quad (53)$$

where

$$p_l = p_b - 2\sigma_b/R_b - 4\mu_l \dot{R}_b/R_b \quad (54)$$

Here, the bubble pressure p_b was evaluated using Eq. (1) together with the bubble mass relation $\rho_b = \rho_{bo}(R_{bo}/R_b)^3$, while the liquid sound speed a_l was obtained from Eq. (5). The commercial software employed a geometric PLIC-VOF method based on a pressure-based PISO solver. To ensure positive pressure within the bubble and to mitigate potential numerical instabilities arising from large temporal pressure variations, several numerical treatments were applied following the approach of Koch et al. [9]:

$$\bar{p}_b = \min(p_b, p_{\min}) \quad (55)$$

$$\tilde{a}_b = \left(\alpha_b/a_b^2 + (1 - \alpha_b)/a_{l,\infty}^2\right)^{-0.5} \quad (56)$$

$$\bar{\rho}_b = \rho_b(t) = \frac{\int_V \rho_b(\mathbf{x}, t) r dV}{\int_V r dV} \quad (57)$$

Here, $\bar{p}_b$ is used to evaluate ρ_b ; $p_{\min}$ denotes the cut-off minimum pressure; $\tilde{a}_b$ is the modified sound speed; $a_{l,\infty} = 1544$ m/s; $\bar{\rho}_b$ represents the averaged bubble density defined in the liquid region adjacent to the bubble liquid–gas interface; and $p_{\min}$ is a predefined artificial constant.

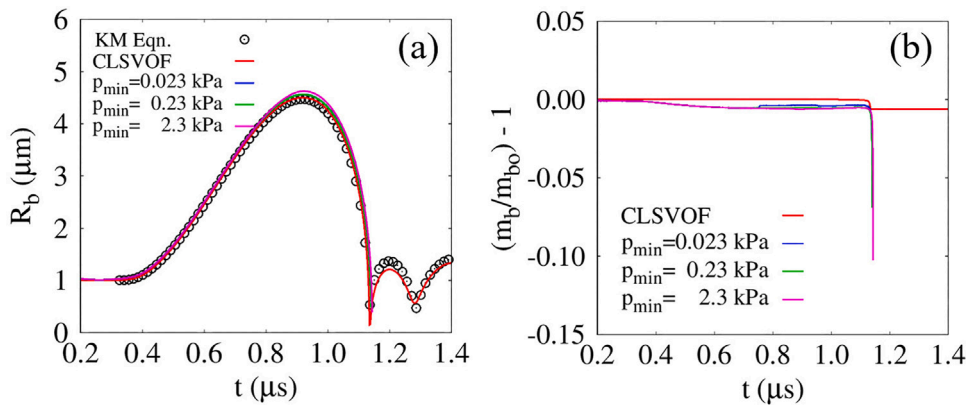


Fig. 8. Single-bubble growth and collapse in ultrasonic fields, compared with the results obtained from the Keller–Miksis equation and a commercial software: (a) bubble-averaged radius R_b and (b) normalized mass variation $m_b/m_{b0} - 1$.

A comparison was made between the time-dependent averaged bubble radius R_b , and the numerical results obtained from the KM equation as well as those from the commercial software for various values of $p_{\min} = 2.3$ kPa, 0.23 kPa, and 0.023 kPa, as shown in Fig. 8(a). During the initial growth–collapse cycle, the present CLSVOF predictions show excellent agreement with the KM solution, owing to the nearly spherical shape of the bubble throughout its motion, as illustrated in Fig. 7.

The agreement between the present results and the KM solution can be reasonably attributed to the fundamental assumptions of the KM equation, which characterizes spherically symmetric bubble motion within a compressible medium driven by a plane pressure wave. The results obtained from the commercial software at $p_{\min} = 0.023$ kPa also show good agreement with those from the present method up to the first bubble collapse. However, as shown in Fig. 8(b), the commercial software exhibited a slight but continuous violation of mass conservation during bubble growth, even before the first collapse. In contrast, the present CLSVOF results showed negligible normalized mass loss $(m_b/m_{b0} - 1)$ throughout the entire process; even after bubble collapse, the total mass deviation remained below 1% of the initial mass. The present results verify the capability of the proposed CLSVOF method to accurately resolve compressible bubble dynamics under ultrasonic excitation.

3.2.2. Ultrasound-driven bubble-bubble interaction

Simulations were performed to investigate two-bubble dynamics under ultrasonic excitation using the present numerical model. Fig. 9 illustrates the response of two identical bubbles ($R_{b01} = R_{b02} = 1 \mu\text{m}$) positioned at $(0, \pm 7 \mu\text{m})$ with a surface-to-surface distance of $l_{bb} = 12 \mu\text{m}$. When the negative pressure pulse reaches the bubbles ($t > 0.32 \mu\text{s}$), both begin to expand, inducing outward radial flow in the surrounding liquid. As both bubbles expand to nearly equal sizes at approximately the same time, symmetric liquid streamlines develop about the centerline ($z = 0 \mu\text{m}$), as shown at $t = 0.740 \mu\text{s}$.

A mirror-like symmetry forms between the two bubbles, consistent with the numerical findings of a previous study [50] which analyzed two-bubble dynamics at high Reynolds numbers using the boundary element method (BEM). Even after the negative pressure phase passes, both continue to grow slightly due to the inertial effect of the surrounding water until $t = 0.930 \mu\text{s}$, when the upper bubble (bubble 1) begins to contract earlier than the lower one (bubble 2). During the contraction phase, mutual attractive interaction arises near the facing surfaces, generating symmetric liquid streamlines, generating symmetric liquid streamlines, as observed at $t = 1.020 \mu\text{s}$. As bubble 1 contracts more rapidly, the strong sink flow that develops around it gradually breaks the flow symmetry, as seen at $t = 1.140 \mu\text{s}$. Following the collapse of bubble 1, bubble 2 undergoes rapid contraction and subsequent collapse between $1.150 \mu\text{s} \leq t \leq 1.160 \mu\text{s}$, producing counter-directed liquid jets in the region between the two bubbles, as shown at $t = 1.179 \mu\text{s}$. These alternating attractive and repulsive interactions throughout the entire process generate a mirror-like flow field, suggesting a close analogy between ultrasound-driven two-bubble dynamics and single-bubble–wall interactions.

Time evolutions of R_b , the averaged bubble internal pressure $p_{b,av}$, and the normalized mass loss $m_b/m_{b0} - 1$ associated with the two-bubble dynamics under ultrasonic excitation are presented in Fig. 10. In comparison with the case of an isolated oscillating bubble, evolution of R_b of bubble 1 exhibits behavior similar to that of a bubble positioned adjacent to a rigid boundary, driven by an acoustic pulse of $P_u = 0.15$ MPa and f_u MHz, with a bubble–wall distance of $l_w = l_{bb}/2 = 6 \mu\text{m}$.

When a reflected pulse from a rigid wall overlaps in phase with the incident pulse, the resulting pressure amplitude nearly doubles due to constructive interference. This similarity between the two-bubble and bubble–wall interactions arises from a virtual rigid-wall effect generated near the midpoint between the two bubbles by the symmetric liquid streamlines, as observed in Fig. 9. This observation agrees with the theoretical explanation provided in Ref. [51], which interprets the wall–bubble system in terms of a mirror-image analogy, where the real bubble behaves as if coupled with its in-phase reflection.

Although the maximum radius $R_{b,max}$ is nearly identical for the two-bubble and bubble–wall configurations, the freely oscillating bubble reaches $R_{b,max} = 4.50 \mu\text{m}$ at $t = 0.918 \mu\text{s}$, which is about 6.4% larger than that obtained for the two-bubble configuration, as shown in Fig. 10(a). Because the smaller $R_{b,max}$ in the two-bubble system leads to a weaker collapse, a similar tendency appears

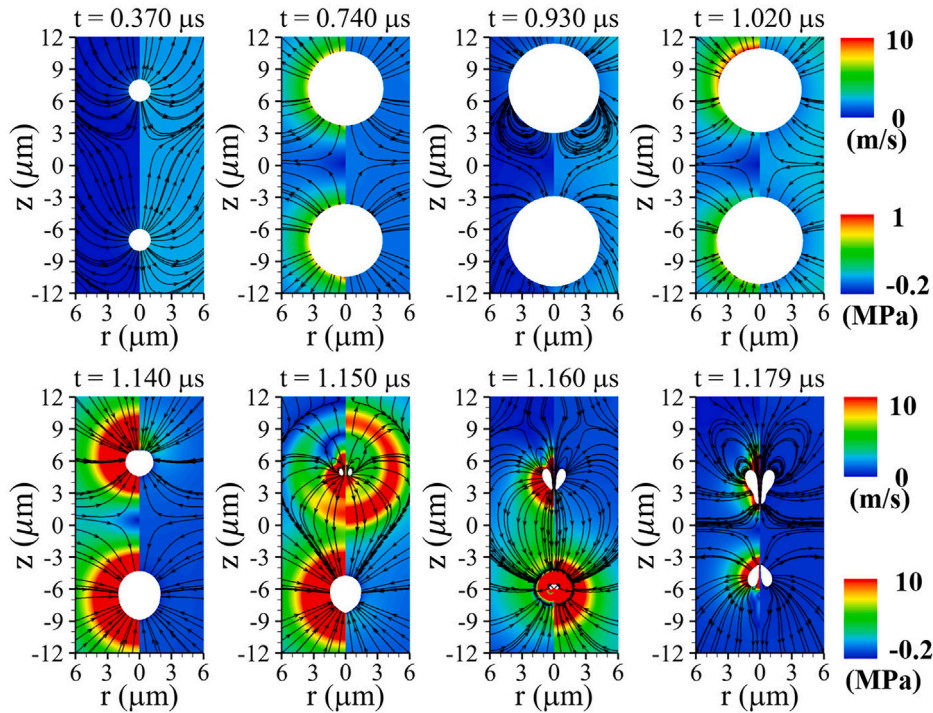


Fig. 9. Liquid velocity (left) and pressure (right) fields with streamlines associated with two-bubble motion at $P_u = 0.3$ MPa and $f_u = 1$ MHz. The white regions denote the bubble interiors.

in the time evolution of $p_{b,av}$, as illustrated in Fig. 10(b). During the collapse phase, the peak value of $p_{b,av}$ for bubble 1 is lower by about 2 orders of magnitude compared with that of a freely oscillating bubble, and is even smaller than that of a bubble near a wall.

These results demonstrate that the collapse intensity in two-bubble dynamics is significantly reduced compared with that of a single freely oscillating bubble, implying that the virtual rigid-wall effect between the two bubbles cannot fully reproduce the behavior of an actual immobile wall. Similar to bubble 1, bubble 2 exhibits comparable oscillatory behavior [Fig. 10(c)], although its R_b is slightly phase-shifted and $R_{b,max}$ is about 0.5% larger than that of bubble 1, consistent with the numerical findings reported in Ref. [52].

The collapse intensity associated with each $R_{b,max}$ also affects the mass loss, as plotted in Fig. 10(d). For a bubble undergoing free oscillation, which attains the largest $R_{b,max}$, a mass loss exceeding 0.5% is observed. In contrast, the two-bubble case exhibits smaller mass losses of approximately 0.04% and 0.12% for bubble 1 and 2, respectively, due to their weaker collapse intensity. Considering that the mass loss remains negligible during the entire bubble growth period, the present CLSVOF method can be regarded as maintaining excellent mass conservation even during the collapse stage.

3.3. Compressible multiphase flow including solid deformation

3.3.1. Ultrasound-driven bubble motion near a spherical solid cell

The compressible CLSVOF approach developed in this study was utilized to compute the coupled dynamics between an air bubble and a deformable solid cell in water. The physical properties were identical to those described in Section 3.2. As commonly assumed in many biological systems, the solid cell with an initial radius $R_{co} = 5R_{bo} = 5 \mu\text{m}$ was considered to have the same properties as water [22,53], i.e., $\mu_s = \mu_l$, $\Pi_s = \Pi_l$, $\gamma_s = \gamma_l$.

The simulations were conducted under axisymmetric conditions within the computational domain defined by $r \leq R_d = 5897 \mu\text{m} \cap z_{d,b} = -5903 \mu\text{m} \leq z \leq z_{d,y} = 508 \mu\text{m}$, as illustrated in Fig. 11(a). A fine uniform mesh with a cell size of $h = 0.1 \mu\text{m}$ was applied near the bubble and the solid tissue ($r \leq 19.2R_{bo}$ and $-25.6R_{bo} \leq y \leq 19.2R_{bo}$), while a coarser non-uniform grid was used in the outer regions.

At the start of the simulation, the solid cell was placed along the z -axis with its centroid located at $z = -7 \mu\text{m}$, and a spherical air bubble was positioned at $z = -7 \mu\text{m} + R_{co} + R_{bo} + l_{bco}$ along the same axis. Here, l_{bco} denotes the initial bubble–cell surface-to-surface distance along the central axis, which was treated as a variable parameter. It should be noted that the elastic shear modulus range considered in this study, $G = 0.01 \text{ MPa} \sim 10 \text{ MPa}$, was selected to cover a broad range of stiffnesses relevant to biological and tissue-mimicking materials [54–56].

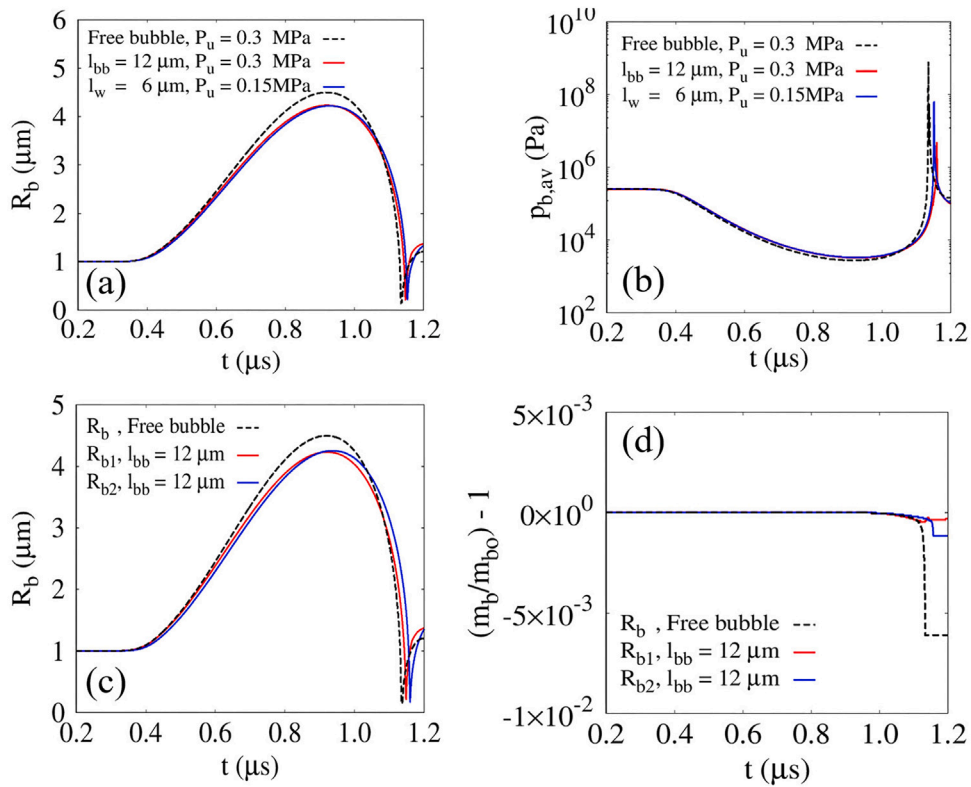


Fig. 10. Two-bubble dynamics in ultrasonic fields compared with those of a single free bubble and a single bubble near a wall: (a) averaged bubble radius R_b ; (b) averaged bubble internal pressure $p_{b,av}$; and (c) normalized mass variation $m_b/m_{bo} - 1$.

Fig. 11(b)–(c) present the evolution of bubble growth and collapse, along with the corresponding liquid velocity and pressure fields, for $l_{bo} = 7 \mu\text{m}$ and $G = 0.1 \text{ MPa}$. When the incident pressure pulse reaches the bubble at $t > 0.32 \mu\text{s}$, bubble expansion begins, generating a radially outward flow that exerts a compressive force on the nearby solid cell. Bubble expansion persists even after the ambient pressure rises above zero, pushing the adjacent cell outward, as observed at $t = 0.850 \mu\text{s}$, until it attains its maximum radius at $t = 0.920 \mu\text{s}$, consistent with previous numerical observations [53].

During the subsequent contraction phase, the deformed cell surface recovers due to elastic stresses, driving an upward motion of the surrounding water, as depicted at $t = 0.950 \mu\text{s}$ in Fig. 11(c). During bubble contraction, an inward sink flow emerges toward the bubble center, evident at $t = 1.130 \mu\text{s}$, which causes local stretching of the upper cell surface. Because the bubble–cell separation is relatively large, the interaction remains weak, resulting in an almost symmetric flow field around the collapsing bubble at $t = 1.135 \mu\text{s}$.

This slight asymmetry in the flow is insufficient to produce a strong downward liquid jet during collapse and thus does not lead to the formation of a toroidal bubble, as illustrated at $t = 1.136 \mu\text{s}$. The shock wave generated upon collapse propagates radially outward with diminishing intensity and reaches the cell surface at $t = 1.139 \mu\text{s}$.

To examine the grid dependence of the present simulations, a grid refinement study was performed for the coupled bubble collapse and cell deformation problem, as shown in Fig. 12. Three grid resolutions, $\Delta r = 0.2 \mu\text{m}$, $0.1 \mu\text{m}$, and $0.05 \mu\text{m}$, were considered under identical computational conditions. The time evolutions of the averaged bubble radius R_b and cell deformation D_c demonstrate that the results obtained with $\Delta r = 0.1 \mu\text{m}$ are in close agreement with those obtained using the finer grid of $\Delta r = 0.05 \mu\text{m}$. In contrast, the coarser grid of $\Delta r = 0.2 \mu\text{m}$ exhibits slight deviations near the bubble collapse stage. The bubble and cell interface profiles during the first expansion–contraction cycle also show negligible differences between the $\Delta r = 0.1 \mu\text{m}$ and $0.05 \mu\text{m}$ cases. These results indicate that the grid resolution of $\Delta r = 0.1 \mu\text{m}$ used in the main simulations is sufficiently fine to accurately resolve the bubble collapse and the associated cell deformation.

In the present simulations, a time step of $\Delta t = 0.1 \text{ ns}$ was employed, which satisfies the explicit time-step restrictions associated with surface tension and elastic shear effects. It should be noted that the explicit time-step restriction associated with the elastic shear modulus is proportional to $(\rho \Delta r^2 / G)^{1/2}$, indicating that smaller time steps are required for stiffer solids.

3.3.2. Effect of initial bubble surface-to-surface distance

Fig. 13 illustrate the liquid flow fields and bubble–cell dynamics under ultrasonic excitation for different initial surface-to-surface distances, $l_{bo} = 5 \mu\text{m}$ and $3 \mu\text{m}$. For $l_{bo} = 5 \mu\text{m}$, the expansion flow generated by bubble growth produces greater compressive

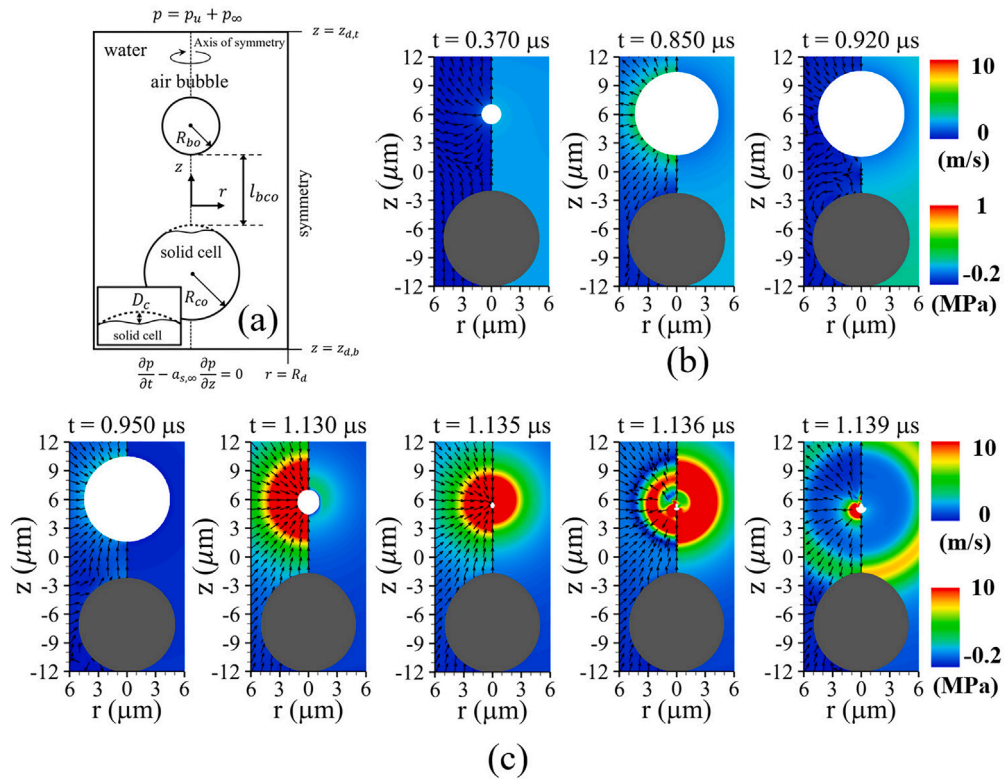


Fig. 11. Ultrasonic bubble motion near a spherical solid cell: (a) schematic of bubble motion and viscoelastic spherical solid deformation; and liquid velocity (left) and pressure (right) fields during (b) the bubble growth period and (c) the contraction and collapse periods, for $P_u = 0.3$ MPa, $f_u = 1$ MHz, $G = 0.1$ MPa, and $l_{bco} = 7 \mu m$. The inset in the lower-left corner of (a) illustrates the definition of the cell deformation D_c along the central axis. The gray and white regions represent the tissue and bubble, respectively.

deformation of the nearby cell at $t = 0.840 \mu s$ and a faster elastic recovery at $t = 0.940 \mu s$, compared with the $l_{bco} = 7 \mu m$ case. During the subsequent contraction phase, the sink flow induced by the collapsing bubble enhances extensional deformation of the solid cell along the central axis, resulting in an elongated shape at $t = 1.130 \mu s$. This demonstrates that a smaller separation distance l_{bco} reduces the resistance to momentum transfer from the bubble motion, thereby amplifying the cell's dynamic response. As the bubble collapses, the annular inflow parallel to the cell surface becomes more pronounced at $t = 1.134 \mu s$, leading to lateral deformation of the bubble. After collapse ($t \geq 1.139 \mu s$), the slightly split bubble rapidly coalesces and re-expands.

When l_{bco} is further reduced to $3 \mu m$, which is smaller than the maximum bubble expansion size ($l_{bco} < R_{b,max}$), bubble–cell interaction intensifies, as illustrated in Fig. 13(b). During bubble expansion, the separation distance progressively decreases, causing pronounced cell deformation and strong elastic restoring stresses at $t = 0.830 \mu s$, while a thin liquid film develops beneath the expanding bubble. The thinning liquid film reaches its minimum thickness at $t = 0.940 \mu s$, enhancing the transmission of repulsive forces and liquid momentum from the cell to the bubble surface. This strong coupling produces alternating attractive and repulsive interactions, leading to severe stretching of the cell and the formation of a droplet-like shape, as also observed in earlier numerical studies [57]. The bubble itself becomes axially elongated as the interaction strengthens.

The intensity of this interaction depends on l_{bco} and is most pronounced along the central axis due to the geometry of the spherical cell. Prior to collapse ($t = 1.134 \mu s$), the enhanced annular inflow causes the bubble to assume a mushroom-like shape, consistent with previous experimental and numerical observations [26,58]. The lateral inflow momentum is subsequently redirected along the axis after collapse, resulting in bubble splitting, as observed at $t = 1.139 \mu s$. The annular inflow becomes increasingly dominant as the cell–bubble interaction strengthens.

The quantitative effects of l_{bco} on R_b , $p_{b,av}$ and cell deformation D_c are plotted in Fig. 14. Here, D_c is defined as the axial displacement of the cell surface along the central axis ($r = 0$) from the equilibrium spherical position, as indicated by the dashed outline in the lower-left inset of Fig. 11(a), following the definition used in Ref. [53]. As l_{bco} decreases from $7 \mu m$ to $3 \mu m$, $R_{b,max}$ remains nearly unchanged. However, for $l_{bco} = 3 \mu m$, the maximum rebound bubble radius $R_{b,max2} = 1.09 \mu m$ at $t = 1.196 \mu s$ is approximately 9.2% smaller than in the other cases, owing to increased viscous damping during rebound of the split bubble [13(b)].

For $l_{bco} > 5 \mu m$, $R_{b,max}$ and $R_{b,max2}$ show little variation, and similar trends appear in $p_{b,av}$ [Fig. 14(b)]. While differences in $R_{b,max}$ and $p_{b,av}$ remain negligible up to the first collapse, cell deformation D_c exhibits a marked dependence on l_{bco} [Fig. 14(c)]. As l_{bco} decreases from $7 \mu m$ to $5 \mu m$, the first minimum deformation $D_{c,min1}$ increases by approximately 76%, from $-0.157 \mu m$ to $-0.276 \mu m$.

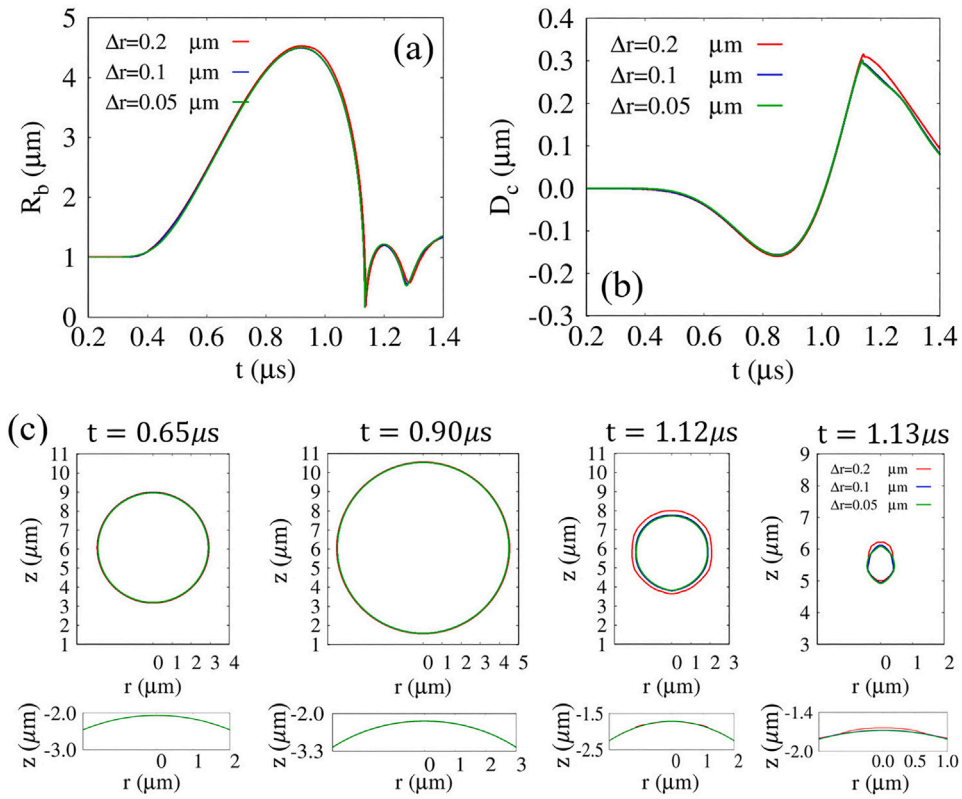


Fig. 12. Grid refinement study for the coupled bubble collapse and cell deformation problem at $P_u = 0.3$ MPa, $f_u = 1$ MHz, $G = 0.1$ MPa, and $l_{bco} = 7$ μm: (a) time evolution of the averaged bubble radius R_b , (b) time evolution of the cell deformation D_c , and (c) bubble (upper) and cell (lower) interface profiles during the first expansion-contraction cycle for $\Delta r = 0.2, 0.1$, and 0.05 μm.

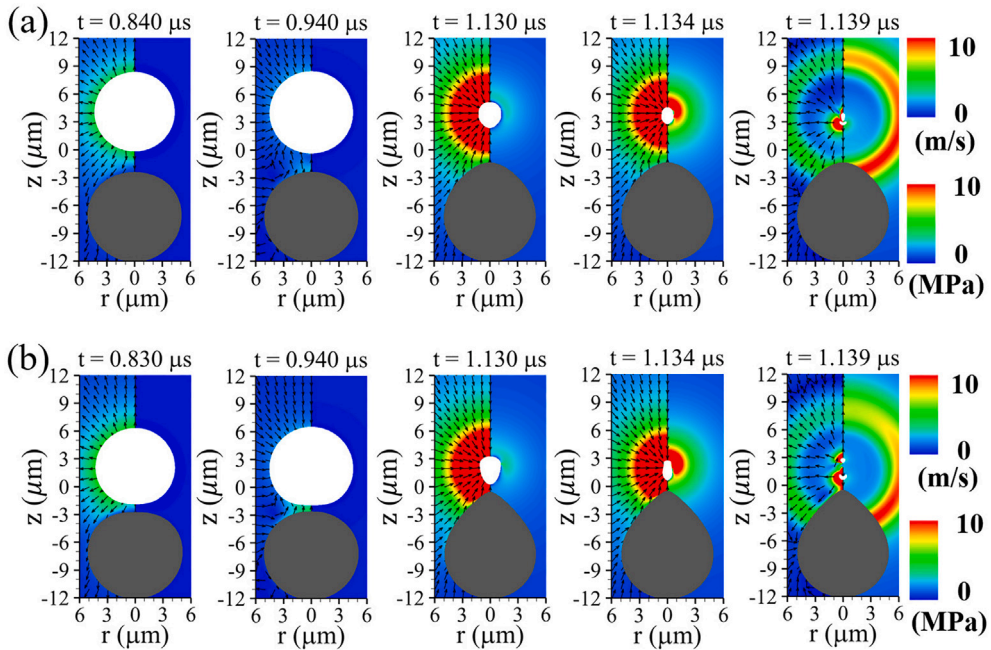


Fig. 13. Ultrasonic bubble motion near a spherical solid cell and the corresponding liquid velocity (left) and pressure (right) fields for (a) $l_{bco} = 5$ μm and (b) $l_{bco} = 3$ μm. The gray and white regions represent the tissue and bubble, respectively.

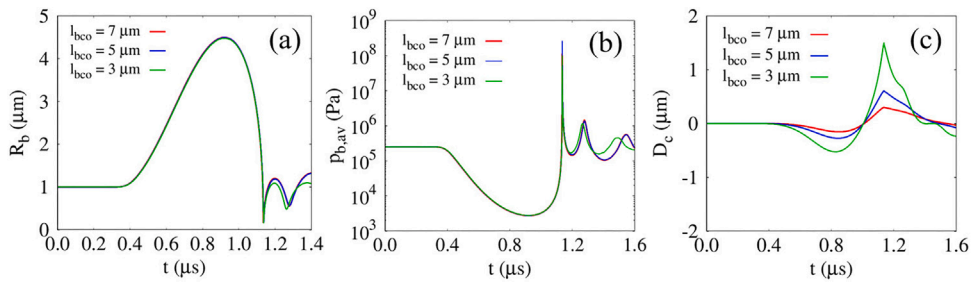


Fig. 14. Effect of the initial surface-to-surface distance l_{bco} on bubble-cell interactions for $G = 0.1$ MPa: (a) averaged bubble radius R_b ; (b) averaged bubble internal pressure $p_{b,av}$; and (c) cell deformation D_c .

For $l_{bco} = 3 \mu\text{m}$, $D_{c,min1} = -0.527 \mu\text{m}$ occurs at $t = 0.825 \mu\text{s}$, about 2.4 times larger than that for the $7 \mu\text{m}$ case due to stronger bubble-cell repulsion. Following elastic rebound and expansion driven by sink flow, the cell attains its first maximum deformation $D_{c,max1} = 1.448 \mu\text{m}$ at $t = 1.139 \mu\text{s}$, which is about 3.8 and 1.4 times greater than for the $l_{bco} = 7 \mu\text{m}$ and $5 \mu\text{m}$ cases, respectively. After collapse, the bubble undergoes several oscillations, while D_c gradually decreases after reaching its peak, irrespective of the bubble motion. These results clearly demonstrate that the ratio $l_{bco}/R_{b,max}$ governs the strength of bubble-cell coupling, which becomes substantially stronger when $l_{bco} < R_{b,max}$.

3.3.3. Effect of elastic shear modulus

Fig. 15 describes the bubble shrinkage and collapse near a spherical cell for $l_{bco} = 3 \mu\text{m}$ at $G = 0.01$ Pa and 10 MPa. For the soft cell case ($G = 0.01$ MPa), the bubble expansion proceeds almost freely, pushing the nearby solid outward with negligible resistance until $t \leq 0.920 \mu\text{s}$, as presented in Fig. 15(a). The onset of bubble contraction nearly coincides with the re-expansion of the compressed cell, reflecting the fluid-like behavior of a low-shear-modulus solid [36]. Because the surrounding solid offers little elastic resistance, the bubble collapses nearly spherically, producing a symmetric flow field at $t = 1.130 \mu\text{s}$. This symmetry causes slight bubble migration toward the cell surface but no jet formation during collapse ($t \geq 1.140 \mu\text{s}$).

In contrast, for the stiff cell ($G = 10$ MPa), the solid retains an almost undeformed spherical shape throughout the expansion stage ($t = 0.920 \mu\text{s}$), generating a very thin liquid film beneath the bubble. This configuration enhances the attractive hydrodynamic interaction, particularly along the central axis ($r = 0$), and results in a highly elongated bubble shape at $t = 1.130 \mu\text{s}$. As the bubble further shrinks, an asymmetric sink flow develops due to the strong hydrodynamic resistance imposed by the stiff cell. Instead of undergoing large deformation, the entire cell translates axially, resulting in the formation of a strong downward liquid jet at $t = 1.137 \mu\text{s}$. The jet drives rapid bubble migration and impact onto the cell surface. Because of the cell's rigidity, noticeable compressive deformation is absent at $t = 1.144 \mu\text{s}$. These observations indicate that the shear modulus not only governs jet formation toward the cell surface but also enhances resistance to deformation, thereby balancing the bubble-induced stresses.

The effects of G on R_b and D_c for $l_{bco} = 3 \mu\text{m}$ are summarized in Fig. 16. As G increases from 0.01 MPa, $R_{b,max}$ changes little, showing a different trend from our previous results for bubble motion near planar tissues [26]. This difference arises because the spherical cell experiences rigid-body translation concurrent with compression, causing outward displacement during bubble expansion. Although $R_{b,max}$ varies minimally with G , the rebound radius $R_{b,max2}$ shows a stronger dependence. At $G = 1$ MPa, the rebounding bubble reaches $R_{b,max2} = 1.606 \mu\text{m}$ at $t = 1.201 \mu\text{s}$, representing the largest rebound among all cases. Throughout the entire growth-collapse cycle, the cell deformation D_c exhibits distinct behaviors with varying G , even before bubble collapse, as shown in Fig. 16(b).

During bubble growth, the soft cell ($G = 0.01$ MPa) behaves in a fluid-like manner and undergoes substantial compression, whereas the stiff cell ($G = 10$ MPa) remains almost undeformed. The intermediate case ($G = 1$ MPa) exhibits pronounced elastic behavior, reaching a first minimum deformation of $|D_{c,min1}| = 0.088 \mu\text{m}$ at $t = 0.693 \mu\text{s}$, corresponding to a 92% reduction compared with the soft-cell case. The cell then recovers earlier under elastic stresses and expands during bubble collapse, achieving a first maximum deformation $D_{c,max1} = 0.439 \mu\text{m}$ at $t = 1.130 \mu\text{s}$.

After reaching its maximum deformation, the cell rapidly compresses and attains a second minimum deformation $|D_{c,min2}| = 5.77 \mu\text{m}$ at $t = 1.253 \mu\text{s}$, which is about 64.5 times larger than $|D_{c,min1}|$. This result indicates that an intermediate cell stiffness promotes both asymmetric flow formation and substantial elastic deformation, producing the largest overall cell response. These findings demonstrate that the elastic shear modulus of the surrounding cell is a key parameter governing bubble-cell interaction dynamics.

4. Conclusions

A mass-conservative CLSVOF framework was developed to simulate compressible multiphase flows with viscoelastic solid deformation. The fully Eulerian formulation, combined with a mass-preserving VOF equation, accurately captures ultrasound-driven

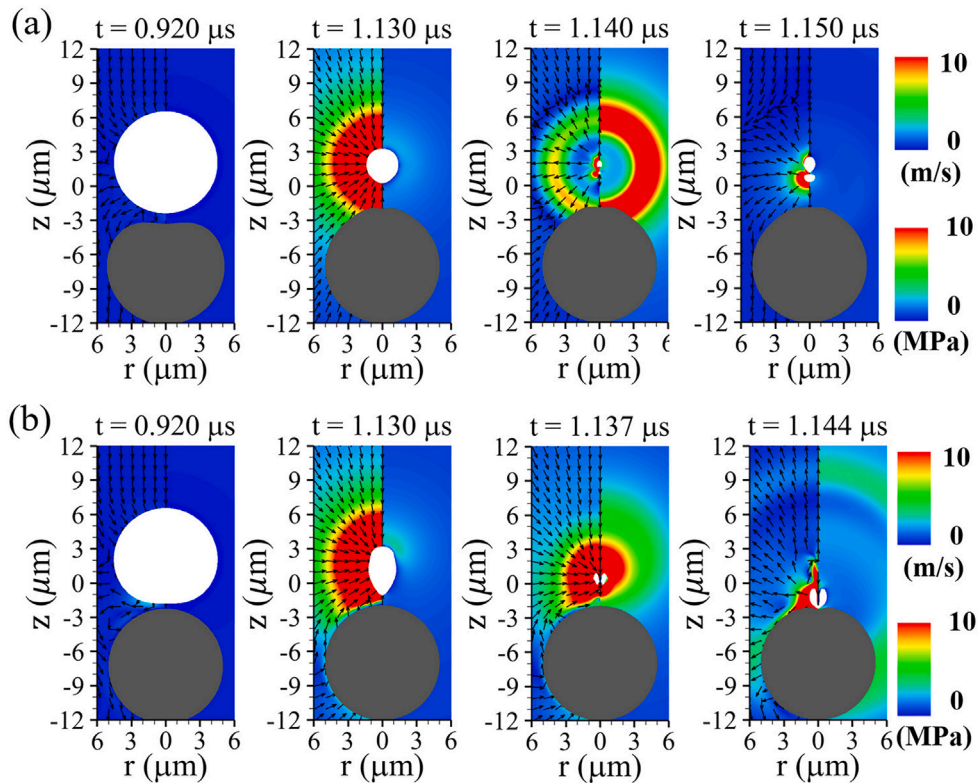


Fig. 15. Liquid velocity (left) and pressure (right) fields for $l_{bco} = 3 \mu\text{m}$ with (a) $G = 0.01 \text{ MPa}$ and (b) $G = 10 \text{ MPa}$. The gray and white regions represent the tissue and bubble, respectively.

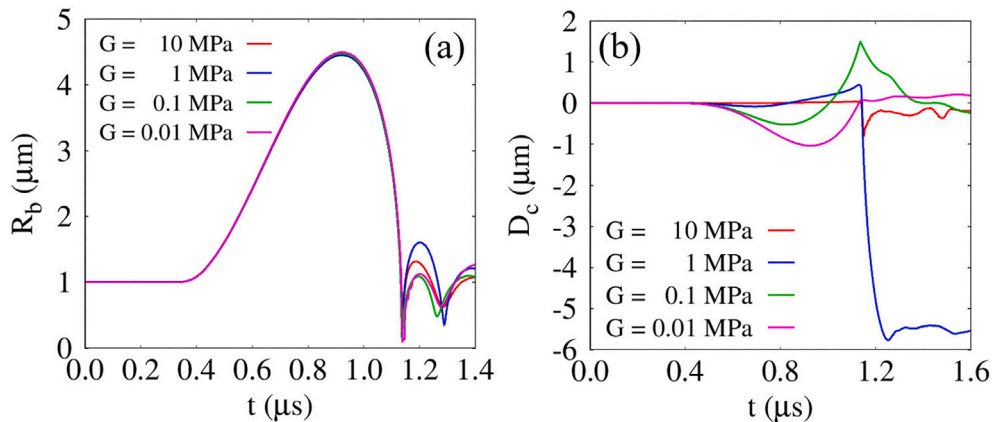


Fig. 16. Effect of the shear modulus on bubble–cell interactions for $l_{bco} = 3 \mu\text{m}$: (a) averaged bubble radius R_b and (b) cell deformation D_c .

bubble dynamics and solid responses. Benchmark validations — including bubble rising, droplet equilibrium, and solid deformation — showed excellent agreement with reference data, maintaining total mass variation below 1% even during bubble collapse. The framework successfully reproduced bubble–bubble and bubble–cell interactions under ultrasonic excitation, revealing that the cell’s elastic modulus governs flow asymmetry and deformation characteristics. These results demonstrate that the proposed method provides a robust and versatile numerical tool for analyzing complex compressible multiphase systems with solid deformation.

CRediT authorship contribution statement

Jaesung Park: Writing – original draft, Software, Conceptualization. **Gihun Son:** Writing – review & editing, Supervision, Methodology, Conceptualization.

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References

- [1] A. Filipić, T. Lukežič, K. Bačnik, M. Ravnikar, M. Ješelnik, T. Košir, M. Petkovšek, M. Zupanc, M. Dular, I.G. Aguirre, Hydrodynamic cavitation efficiently inactivates potato virus Y in water, *Ultrason. Sonochem.* 82 (2022) 105898, <http://dx.doi.org/10.1016/j.ultsonch.2021.105898>.
- [2] K. Kooiman, S. Roovers, S.A. Langeveld, R.T. Kleven, H. Dewitte, M.A. O'Reilly, J.-M. Escoffre, A. Bouakaz, M.D. Verweij, K. Hynynen, et al., Ultrasound-responsive cavitation nuclei for therapy and drug delivery, *Ultrasound Med. Biol.* 46 (6) (2020) 1296–1325, <http://dx.doi.org/10.1016/j.ultrasmedbio.2020.01.002>.
- [3] N. Vyas, Q. Wang, K. Manmi, R. Sammons, S. Kuehne, A. Walmsley, How does ultrasonic cavitation remove dental bacterial biofilm? *Ultrason. Sonochem.* 67 (2020) 105112, <http://dx.doi.org/10.1016/j.ultsonch.2020.105112>.
- [4] R.P. Fedkiw, T. Aslam, B. Merriman, S. Osher, A non-oscillatory Eulerian approach to interfaces in multimaterial flows (the ghost fluid method), *J. Comput. Phys.* 152 (2) (1999) 457–492, <http://dx.doi.org/10.1016/j.jcp.1999.6236>.
- [5] Hu, Khoo, An interface interaction method for compressible multifluids, *J. Comput. Phys.* 198 (1) (2004) 35–64, <http://dx.doi.org/10.1016/j.jcp.2003.12.018>.
- [6] G. Huber, S. Tanguy, J.-C. Béra, B. Gilles, A time splitting projection scheme for compressible two-phase flows. application to the interaction of bubbles with ultrasound waves, *J. Comput. Phys.* 302 (2015) 439–468, <http://dx.doi.org/10.1016/j.jcp.2015.09.019>.
- [7] J. Lee, G. Son, A sharp-interface level-set method for compressible bubble growth with phase change, *Int. Commun. Heat Mass Transf.* 86 (2017) 1–11, <http://dx.doi.org/10.1016/j.icheatmasstransfer.2017.05.016>.
- [8] S. Miller, H. Jasak, D. Boger, E. Paterson, A. Nedungadi, A pressure-based, compressible, two-phase flow finite volume method for underwater explosions, *Comput. & Fluids* 87 (2013) 132–143, <http://dx.doi.org/10.1016/j.compfluid.2013.04.002>.
- [9] M. Koch, C. Lechner, F. Reuter, K. Köhler, R. Mettin, W. Lauterborn, Numerical modeling of laser generated cavitation bubbles with the finite volume and volume of fluid method, using OpenFOAM, *Comput. & Fluids* 126 (2016) 71–90, <http://dx.doi.org/10.1016/j.compfluid.2015.11.008>.
- [10] J. Mifsud, D.A. Lockerby, Y.M. Chung, G. Jones, Numerical simulation of a confined cavitating gas bubble driven by ultrasound, *Phys. Fluids* 33 (12) (2021) <http://dx.doi.org/10.1063/5.0075280>.
- [11] D. Fuster, S. Popinet, An all-mach method for the simulation of bubble dynamics problems in the presence of surface tension, *J. Comput. Phys.* 374 (2018) 752–768, <http://dx.doi.org/10.1016/j.jcp.2018.07.055>.
- [12] Y. Saade, D. Lohse, D. Fuster, A multigrid solver for the coupled pressure-temperature equations in an all-mach solver with VoF, *J. Comput. Phys.* 476 (2023) 111865, <http://dx.doi.org/10.1016/j.jcp.2022.111865>.
- [13] I. Ginzburg, G. Wittum, Two-phase flows on interface refined grids modeled with VOF, staggered finite volumes, and spline interpolants, *J. Comput. Phys.* 166 (2) (2001) 302–335, <http://dx.doi.org/10.1006/jcp.2000.6655>.
- [14] M. Raessi, J. Mostaghimi, M. Bussmann, A volume-of-fluid interfacial flow solver with advected normals, *Comput. & Fluids* 39 (8) (2010) 1401–1410, <http://dx.doi.org/10.1016/j.compfluid.2010.04.010>.
- [15] B. Duret, R. Canu, J. Reveillon, F. Demoulin, A pressure based method for vaporizing compressible two-phase flows with interface capturing approach, *Int. J. Multiph. Flow* 108 (2018) 42–50, <http://dx.doi.org/10.1016/j.ijmultiphaseflow.2018.06.022>.
- [16] X. Ma, B. Huang, Y. Li, Q. Chang, S. Qiu, Z. Su, X. Fu, G. Wang, Numerical simulation of single bubble dynamics under acoustic travelling waves, *Ultrason. Sonochem.* 42 (2018) 619–630, <http://dx.doi.org/10.1016/j.ultsonch.2017.12.021>.
- [17] I. Chakraborty, Numerical modeling of the dynamics of bubble oscillations subjected to fast variations in the ambient pressure with a coupled level set and volume of fluid method, *Phys. Rev. E* 99 (4) (2019) 043107, <http://dx.doi.org/10.1103/PhysRevE.99.043107>.
- [18] G.L. Chahine, A. Kapahi, J.-K. Choi, C.-T. Hsiao, Modeling of surface cleaning by cavitation bubble dynamics and collapse, *Ultrason. Sonochem.* 29 (2016) 528–549, <http://dx.doi.org/10.1016/j.ultsonch.2015.04.026>.
- [19] C. Kauffhold, F. Pöhl, S. Mottyll, R. Skoda, W. Theisen, Numerical simulation of the deformation behavior of metallic materials under cavitation induced load in the incubation period, *Wear* 376 (2017) 1138–1146, <http://dx.doi.org/10.1016/j.wear.2016.11.024>.
- [20] R. Firly, K. Inaba, F. Triawan, K. Kishimoto, H. Nakamoto, Numerical study of impact phenomena due to cavitation bubble collapse on metals and polymers, *Eur. J. Mech. B-Fluids* 101 (2023) 257–272, <http://dx.doi.org/10.1016/j.euromechflu.2023.06.004>.
- [21] Y. Sunayama, J.F.T. Rabago, M. Kimura, Comoving mesh method for multi-dimensional moving boundary problems: Mean-curvature flow and stefan problems, *Math. Comput. Simulation* 221 (2024) 589–605, <http://dx.doi.org/10.1016/j.matcom.2024.03.020>.
- [22] H. Zhao, J.B. Freund, R.D. Moser, A fixed-mesh method for incompressible flow–structure systems with finite solid deformations, *J. Comput. Phys.* 227 (6) (2008) 3114–3140, <http://dx.doi.org/10.1016/j.jcp.2007.11.019>.
- [23] S. Li, X. Gong, K. Sugiyama, J. Wu, H. Huang, S. Takagi, A full Eulerian fluid-membrane coupling method with a smoothed volume-of-fluid approach, *Commun. Comput. Phys.* 12 (2) (2012) 544–576, <http://dx.doi.org/10.4208/cicp.141210.110811s>.
- [24] L. Tao, X.-L. Deng, A new and improved cut-cell-based sharp-interface method for simulating compressible fluid elastic–perfectly plastic solid interaction, *Math. Comput. Simulation* 171 (2020) 246–263, <http://dx.doi.org/10.1016/j.matcom.2019.05.010>.
- [25] E. Koukas, A. Papoutsakis, M. Gavaies, Numerical investigation of shock-induced bubble collapse dynamics and fluid–solid interactions during shock-wave lithotripsy, *Ultrason. Sonochem.* 95 (2023) 106393, <http://dx.doi.org/10.1016/j.ultsonch.2023.106393>.
- [26] J. Park, G. Son, Numerical investigation of acoustic cavitation and viscoelastic tissue deformation, *Ultrason. Sonochem.* 102 (2024) 106757, <http://dx.doi.org/10.1016/j.ultsonch.2024.106757>.
- [27] G. Son, N. Hur, A coupled level set and volume-of-fluid method for the buoyancy-driven motion of fluid particles, *Numer. Heat Transf. B-Fundam.* 42 (6) (2002) 523–542, <http://dx.doi.org/10.1080/10407790260444804>.
- [28] K. Sugiyama, S. Li, S. Takeuchi, S. Takagi, Y. Matsumoto, A full Eulerian finite difference approach for solving fluid–structure coupling problems, *J. Comput. Phys.* 230 (3) (2011) 596–627, <http://dx.doi.org/10.1016/j.jcp.2010.09.032>.

- [29] S. Osher, J.A. Sethian, Fronts propagating with curvature-dependent speed: Algorithms based on hamilton-Jacobi formulations, *J. Comput. Phys.* 79 (1) (1988) 12–49, [http://dx.doi.org/10.1016/0021-9991\(88\)90002-2](http://dx.doi.org/10.1016/0021-9991(88)90002-2).
- [30] H. Kumar, S. Rakshit, Multi-material topology optimization using isogeometric method based reaction–diffusion level set techniques, *Math. Comput. Simulation* 233 (2025) 530–552, <http://dx.doi.org/10.1016/j.matcom.2025.02.010>.
- [31] K. Kalumuck, R. Duraiswami, G. Chahine, Bubble dynamics fluid-structure interaction simulation by coupling fluid BEM and structural FEM codes, *J. Fluids Struct.* 9 (8) (1995) 861–883, <http://dx.doi.org/10.1006/jfls.1995.1049>.
- [32] P. Koukouvini, M. Gavaises, O. Supponen, M. Farhat, Numerical simulation of a collapsing bubble subject to gravity, *Phys. Fluids* 28 (3) (2016) <http://dx.doi.org/10.1063/1.4944561>.
- [33] T. Trummer, S.J. Schmidt, N.A. Adams, Numerical investigation of non-condensable gas effect on vapor bubble collapse, *Phys. Fluids* 33 (9) (2021) <http://dx.doi.org/10.1063/5.0062399>.
- [34] L. Bergamasco, D. Fuster, Oscillation regimes of gas/vapor bubbles, *Int. J. Heat Mass Transfer* 112 (2017) 72–80, <http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.04.082>.
- [35] J. Park, G. Son, Numerical investigation of acoustic droplet vaporization and tissue deformation, *Math. Comput. Simulation* 225 (2024) 412–429, <http://dx.doi.org/10.1016/j.matcom.2024.05.030>.
- [36] J. Park, G. Son, A level-set method for ultrasound-driven bubble motion and tissue deformation, *Commun. Nonlinear Sci. Numer. Simul.* 128 (2024) 107619, <http://dx.doi.org/10.1016/j.cnsns.2023.107619>.
- [37] M. Sussman, K.M. Smith, M.Y. Hussaini, M. Ohta, R. Zhi-Wei, A sharp interface method for incompressible two-phase flows, *J. Comput. Phys.* 221 (2) (2007) 469–505, <http://dx.doi.org/10.1016/j.jcp.2006.06.020>.
- [38] B. Van Leer, Towards the ultimate conservative difference scheme. IV. A new approach to numerical convection, *J. Comput. Phys.* 23 (3) (1977) 263–275, [http://dx.doi.org/10.1016/0021-9991\(77\)90095-X](http://dx.doi.org/10.1016/0021-9991(77)90095-X).
- [39] C. Baranger, N. Hérouard, J. Mathiaud, L. Mieussens, Numerical boundary conditions in finite volume and discontinuous Galerkin schemes for the simulation of rarefied flows along solid boundaries, *Math. Comput. Simulation* 159 (2019) 136–153, <http://dx.doi.org/10.1016/j.matcom.2018.11.011>.
- [40] C.-D. Munz, S. Roller, R. Klein, K.J. Geratz, The extension of incompressible flow solvers to the weakly compressible regime, *Comput. & Fluids* 32 (2) (2003) 173–196, [http://dx.doi.org/10.1016/S0045-7930\(02\)00010-5](http://dx.doi.org/10.1016/S0045-7930(02)00010-5).
- [41] A. Urbano, M. Bibal, S. Tanguy, A semi implicit compressible solver for two-phase flows of real fluids, *J. Comput. Phys.* 456 (2022) 111034, <http://dx.doi.org/10.1016/j.jcp.2022.111034>.
- [42] M. Bibal, M. Deferrez, S. Tanguy, A. Urbano, A compressible solver for two phase-flows with phase change for bubble cavitation, *J. Comput. Phys.* 500 (2024) 112750, <http://dx.doi.org/10.1016/j.jcp.2023.112750>.
- [43] J. Poblador-Ibanez, N. Valle, B.J. Boersma, A momentum balance correction to the non-conservative one-fluid formulation in boiling flows using volume-of-fluid, *J. Comput. Phys.* 524 (2025) 113704, <http://dx.doi.org/10.1016/j.jcp.2024.113704>.
- [44] N. Kwatra, J. Su, J.T. Grétarsson, R. Fedkiw, A method for avoiding the acoustic time step restriction in compressible flow, *J. Comput. Phys.* 228 (11) (2009) 4146–4161, <http://dx.doi.org/10.1016/j.jcp.2009.02.027>.
- [45] M. Jemison, M. Sussman, M. Arienti, Compressible, multiphase semi-implicit method with moment of fluid interface representation, *J. Comput. Phys.* 279 (2014) 182–217, <http://dx.doi.org/10.1016/j.jcp.2014.09.005>.
- [46] M. Rudman, A volume-tracking method for incompressible multifluid flows with large density variations, *Internat. J. Numer. Methods Fluids* 28 (2) (1998) 357–378, [http://dx.doi.org/10.1002/\(SICI\)1097-0363\(19980815\)28:2<357::AID-FLD750>3.0.CO;2-D](http://dx.doi.org/10.1002/(SICI)1097-0363(19980815)28:2<357::AID-FLD750>3.0.CO;2-D).
- [47] P. He, R. Qiao, A full-Eulerian solid level set method for simulation of fluid–structure interactions, *Microfluid. Nanofluid.* 11 (5) (2011) 557–567, <http://dx.doi.org/10.1007/s10404-011-0821-6>.
- [48] H. Patel, S. Das, J. Kuipers, J. Padding, E. Peters, A coupled volume of fluid and immersed boundary method for simulating 3D multiphase flows with contact line dynamics in complex geometries, *Chem. Eng. Sci.* 166, 28–41, <http://dx.doi.org/10.1016/j.ces.2017.03.012>.
- [49] K. Kobayashi, T. Kodama, H. Takahira, Shock wave–bubble interaction near soft and rigid boundaries during lithotripsy: numerical analysis by the improved ghost fluid method, *Phys. Med. Biol.* 56 (19) (2011) 6421, <http://dx.doi.org/10.1088/0031-9155/56/19/016>.
- [50] R. Han, A. Zhang, Y. Liu, Numerical investigation on the dynamics of two bubbles, *Ocean Eng.* 110 (2015) 325–338, <http://dx.doi.org/10.1016/j.oceaneng.2015.10.032>.
- [51] A.A. Doinikov, L. Aired, A. Bouakaz, Dynamics of a contrast agent microbubble attached to an elastic wall, *IEEE Trans. Med. Imaging* 31 (3) (2011) 654–662, <http://dx.doi.org/10.1109/TMI.2011.2174647>.
- [52] X. Huang, Q.-X. Wang, A.-M. Zhang, J. Su, Dynamic behaviour of a two-microbubble system under ultrasonic wave excitation, *Ultrason. Sonochem.* 43 (2018) 166–174, <http://dx.doi.org/10.1016/j.ultsonch.2018.01.012>.
- [53] X. Guo, C. Cai, G. Xu, Y. Yang, J. Tu, P. Huang, D. Zhang, Interaction between cavitation microbubble and cell: A simulation of sonoporation using boundary element method (BEM), *Ultrason. Sonochem.* 39 (2017) 863–871, <http://dx.doi.org/10.1016/j.ultsonch.2017.06.016>.
- [54] E. Vlasisavljevic, K.-W. Lin, A. Maxwell, M.T. Warnez, L. Mancia, R. Singh, A.J. Putnam, B. Fowlkes, E. Johnsen, C. Cain, et al., Effects of ultrasound frequency and tissue stiffness on the histotripsy intrinsic threshold for cavitation, *Ultrasound Med. Biol.* 41 (6) (2015) 1651–1667, <http://dx.doi.org/10.1016/j.ultrasmedbio.2015.01.028>.
- [55] E. Song, Y. Huang, N. Huang, Y. Mei, X. Yu, J.A. Rogers, Recent advances in microsystem approaches for mechanical characterization of soft biological tissues, *Microsyst. Nanoeng.* 8 (1) (2022) 77, <http://dx.doi.org/10.1038/s41378-022-00412-z>.
- [56] A. Wahlsten, A. Stracuzzi, I. Lüchtefeld, G. Restivo, N. Lindenblatt, C. Giampietro, A.E. Ehret, E. Mazza, Multiscale mechanical analysis of the elastic modulus of skin, *Acta Biomater.* 170 (2023) 155–168, <http://dx.doi.org/10.1038/s41378-022-00412-z>.
- [57] J. Zevnik, M. Dular, Cavitation bubble interaction with compliant structures on a microscale: A contribution to the understanding of bacterial cell lysis by cavitation treatment, *Ultrason. Sonochem.* 87 (2022) 106053, <http://dx.doi.org/10.1016/j.ultsonch.2022.106053>.
- [58] A. Sieber, D. Preso, M. Farhat, Cavitation bubble dynamics and microjet atomization near tissue-mimicking materials, *Phys. Fluids* 35 (2) (2023) 027101, <http://dx.doi.org/10.1063/5.0136577>.